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SWITCHED FUZZY SYSTEMS: NEW DIRECTIONS FOR INTELLIGENT CONTROL AND DECISION

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This research was carried out in co-operation with
Dr Hong Yang and Prof. Jun Zhao along with several doctoral students at Northeastern University, Shenyang, People's Rep. of China, and at SS Cyril and Methodius University Rep. of Macedonia, during a decade of

Switched Fuzzy Systems: New Directions for Intelligent Control and Decision

OUTLINE OF THE PRESENTATION

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- I. Introduction
- II. Novel Models of Switched Fuzzy Systems
- III. New Stability Results
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Switched Fuzzy Systems: New Directions for Intelligent Control and Decision

PROLOGUE

- It is my humble opinion, the field of Systems Sciences encompassing Communications, Computer and Control Sciences is the **FIFTH fundamental science** next to Biology, Chemistry, Mathematics, and Physics.
- Whereas Biology, Chemistry, and Physics are dominated by paradigms of ENEGRY and MATTER, the first two fundamental and Tangible quantities of the real-world amenable to our natural bio-physical sensors, Systems Sciences similar to Mathematics are dominated by the third real-world quantity of INFORMATION, which requires sensors acompanied with a certain comprehension capacity hence involves intelligence.
- * * * * * * *
- Ages ago in the world's first book of military treatise THE ART OF WAR, i.e. **Military Command Control and Supervision,** the author Sun Tzu has had observed the philosophy of Confucianism. Thinking and intelligent decision making is crucial issue involved in command and control, which should follow Confucious (Cong-fu Tze) wisdom:
- *"LEARNING WITHOUT THINKING IS LABOUR LOST, USELESS. HOWEVER, THINKING WITHOUT LEARNING IS PERILOUS, DANGEROUS."*
- * * * * * * *

Switched Fuzzy Systems: New Directions for Intelligent Control and Decision

- **PROLOGUE**

- In due time, while serving on the Executive Council of European Science Foundation (ESF), I undertook initiative for an European project called COSY – Control of Complex Systems, and a number of European colleagues whom I co-operated within the IFAC embraced the idea. This ESF Scientific Programme took place during 1995-1999 under the leadership of Professors Karl J. Astroem and Manfred Thoma. Me too, I was invited into ESF-COSY as guest-member of its Steering Committee, and took part (with the ESF funding) into this large Pan-European project along with my post-doc fellow Dr Yuanwei Jing and my graduate students. In 2011, the ETAI Society of Macedonia organized a special COSY conference, and the monograph ***"Complex Systems: Synergies between Control, Communications and Computing"***, G. M. Dimirovski, Editor, Springer International Publication, Cham, CH, 2016 (Studies in Systems, Decision and Control, Volume 55) has appeared.

- During the last 20 years, departing from the above background thinking of mine, my research interests and international academic co-operation have been focused on complex systems control and synergies with computational intelligence and communications.

Switched Fuzzy Systems: New Directions for Intelligent Control and Decision

PROLOGUE

- In fact, I focused on **the synergy** of math-analytical and computational-intelligence methodologies in control and decision systems as well as their applications. In doing so, I did try and still am trying to go ahead with the following tasks:

- $\Rightarrow$ Synergy oriented methods of fuzzy-neural and fuzzy-Petri system representation models and control algorithms.
- $\Rightarrow$ Complex interconnected similarity and symmetry dynamical systems.
- $\Rightarrow$ Switched systems and switching in the synthesis of control and decision algorithms.

- Thus, in co-operation with my Chinese partners Jun Zhao and Yuanwei Jing, I came to understand considerably the SYNERGIES such as in:
- \- Switching in Systems and Control and of Fuzzy-logic in Systems and Control.
- \- Control and Supervision Problems in Network-like Systems, and in particular in some types of communication networks and of computer networks.

I. Introduction

- The large class of switched systems has attracted extensive research during the last couple of decades both as such and also in conjunction of the even larger class of hybrid systems, e.g. see [2], [8], [13], [14], and [22]. For, these systems have a wide range of potential applications. For instance, such systems are widely used in the multiple operating point control systems, the systems of power transmission and distribution, constrained robotic systems, intelligent vehicle highway systems, etc.
- Basically, a switched system consists of a family of continuous-time or discrete-time subsystems and a switching rule law that governs and orchestrates the switching among them.

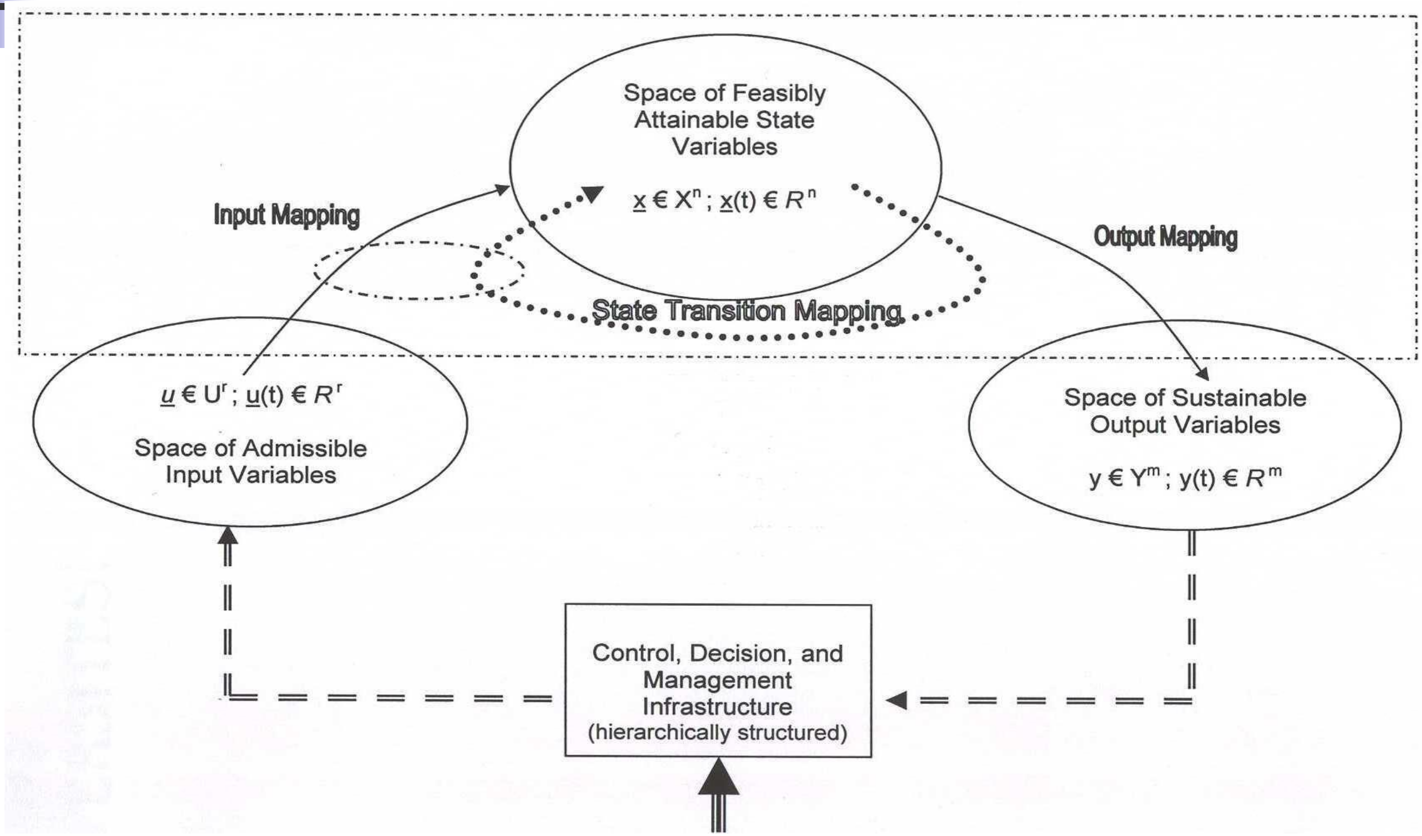
- Recently switched systems have been extended further to encompass switched fuzzy systems too [12], [14], [20] following the advances in fuzzy sliding mode control [4], [9], [11]. Though for long time it was well known the ideal relay switching is a time optimal control law [17]. ***A switched fuzzy system involves fuzzy systems among its sub-systems.*** This extension emanated out of the remarkable developments in theory, applications, and the industrial implementations of fuzzy control systems, e.g. see [1], [16], [18], [19], and [24], with a curcial role of Lyapunov stability theory.

I. Introduction

- It appeared, the class of switched fuzzy systems can describe more precisely both continuous and discrete dynamics as well as their interactions in complex real-world systems. In comparison to either switched or fuzzy control systems, still few stability results on switched fuzzy control systems can be found in the literature. For the continuous-time case, in [12] a combination of hybrid systems and fuzzy multiple model systems was described and an idea of the fuzzy switched hybrid control was put forward. For the discrete-time case, in [5], a fuzzy model whose subsystems are switched systems was described. In this model switching takes place simply based on state variables or time. Subsequently the same authors gave some extensions to output [6] and to guaranteed-cost [7] control designs.

- **In here, an innovated representation modelling of continuous-time and discrete-time switched fuzzy systems is proposed. Sufficient conditions for asymptotic stability are derived by using the method of single *LYAPUNOV* function and the parallel distributed compensation (PDC) fuzzy controller scheme as well as the stabilizing state–dependent switching laws.**

I. Introduction



- Fig.1 Mathematical conceptualization of general dynamic processes and control systems in terms that are consistent with basic natural laws and engineering constraints

II. Novel Models of Switched Fuzzy Systems

- In this paper, only Takagi-Sugeno (*T-S*) fuzzy systems representing the category of data driven models are considered. This representation differs from the existing ones in the literature cited: each sub-system is a fuzzy system hence defining an entire class of switched fuzzy systems. This class inherits some essential features of hybrid systems [2], [14] and retains all the information and knowledge representation capacity of fuzzy systems [16].

- *A. The Continuous-time Case*

- Consider the continuous fuzzy model that involves $N_{\sigma(t)}$ rules of the type as follows:

(1) $$R^{l}_{\sigma(t)} : If\ \ \xi_1\ is\ M^{l}_{\sigma(t)1} \cdots and\ \xi_p\ is\ M^{l}_{\sigma(t)p},\ Then\ \dot{x} = A_{\sigma(t)l}x(t) + B_{\sigma(t)l}u_{\sigma(t)}(t) \qquad l = 1, 2, ..., N_{\sigma(t)}$$

where

$$\sigma : R_{+} \rightarrow M = \{1, 2, \cdots, m\}$$

is a piecewise constant function and it is representing the *switching* signal.

II. Novel Models of Switched Fuzzy Systems

- In rule-based model (1), $R^{l}_{\sigma(t)}$ denotes the *l-th* fuzzy inference rule, $N_{\sigma(t)}$ is the
- number of inference rules, $u(t)$ is input variable, $x(t)=[x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T \in R^n$
- represents the state variables.
- Vector $\xi=[\xi_1\ \xi_2\ \cdots\ \xi_p]$ represents the vector of rule antecendent (premise)
- variables. In the linear dynamic model of the rule consequent, the matrices
- $A_{\sigma(t)l} \in R^{n\times n}$ and $B_{\sigma(t)l} \in R^{n\times p}$ are assumed to have the appropriate dimensions.
- The *i-th* fuzzy subsystem can be represented as follows:

(2) $R^l_i : If\ \xi_1\ is\ M^l_{i1} \cdots and\ \xi_p\ is\ M^l_{ip},\ Then\ \dot{x}(t)=A_{il}x(t)+B_{il}u_i(t) \quad l=1,2,...,N_i,\ i=1,2,...,m.$

Thus the overall model of the *i-th* fuzzy sub-system is described by means of the equation

(3)
$$\dot{x}(t)=\sum_{l=1}^{N_i}\eta_{il}(\xi(t))\left(A_{il}x(t)+B_{il}u_i(t)\right)$$

Along with

II. Novel Models of Switched Fuzzy Systems

- (4-a) – (4-b) $\eta_{il}(t)=\dfrac{\prod_{\rho=1}^{n}\mu_{M_{\rho}^{l}(t)}}{\sum_{l=1}^{N_i}\prod_{\rho=1}^{n}\mu_{M_{\rho}^{l}(t)}}$, and $0\le\eta_{il}(t)\le 1,\ \sum_{l=1}^{N_i}\eta_{il}(t)=1.$
- Where in the above model (3)-(4), the symbol $\mu_{M_{\rho}^{l}(t)}$ denotes the
- membership function of the fuzzy state variable x_{ρ} that belongs to fuzzy
- subset M_{ρ}^{l} .

- *B. The Discrete-time Case*
- Similarly, we can define the discrete switched T-S fuzzy model including
- $N_{\sigma(k)}$ pieces of rules. Namely:
- (5) $R_{\sigma(k)}^{l}: If\ \xi_1\ is\ M_{\sigma(k)1}^{l}\cdots and\ \xi_p\ is\ M_{\sigma(k)p}^{l}, Then\ x(k+1)=A_{\sigma(k)l}x(k)+B_{\sigma(k)l}u_{\sigma(k)}(k)$
  $l=1,2,...,N_{\sigma(k)}$
- where $\sigma(k):\{0,1,\cdots\}\to\{1,2,\cdots,m\}$ represents switching signal $\{0,Z_{+}\}\to\{1,2,\cdots,m\}$

II. Novel Models of Switched Fuzzy Systems

- The *i-th* fuzzy sub-system can be represented as follows:
- (6) $$R_i^l: \ If \ \xi_1 \ is \ M_{i1}^l \cdots and \ \xi_p \ is \ M_{ip}^l, \ Then \ x(k+1) = A_{il}x(k) + B_{il}u_i(k)$$ $$l = 1, 2, \ldots, N_i, \ i = 1, 2, \cdots, m.$$
- Therefore the overall model of the i-th fuzzy sub-system is described by means of the equation
- (7) $$x(k+1) = \sum_{l=1}^{N_i} \eta_{il}(k)\big(A_{il}x(k) + B_{il}u_i(k)\big)$$
- along with
- (8-a)-(8-b) $$\eta_{il}(k) = \frac{\prod_{\rho=1}^{n} \mu_{M_\rho^l(k)}}{\sum_{l=1}^{N_i}\prod_{\rho=1}^{n} \mu_{M_\rho^l(k)}},$$ and $$0 \le \eta_{il}(k) \le 1, \quad \sum_{l=1}^{N_i} \eta_{il}(k) = 1.$$
- Next we discuss the new stability results.

III. New Stability Results

- First the respective definition for quadratic asymptotic stability of switched nonlinear systems, e.g. see [2], [14], [25], [26], and a related lemma in conjunction with stability analysis are recalled.

- *Definition 3.1.* The system (1) is said to be quadratic asymptotic stable
- if there exist a positive definite matrix *P* and a state-dependent
- switching law $\sigma = \sigma(x)$ such that quadratic Lyapunov function
- $V(x(t)) = x^T(t)P\,x(t)$ satisfies $\frac{d}{dt}V(x(t)) < 0$ for any $x(t) \neq 0$ along the
- system state trajectory from arbitrary initial conditions.

- Remark 3.1. An analogous definition can be stated for the discrete-time case of system (5).

III. New Stability Results

- *Lemma 3.1.* Let $a_{ij_i}(1 \le i \le m, 1 \le j_i \le N_i)$ be a set of constants satisfying

$$\sum_{i=1}^{m} a_{ij_i} < 0, \ \forall 1 \le j_i \le N_i.$$

- Then, there exists at least one index *i* such that

$$a_{ij_i} < 0, \ 1 \le j_i \le N_i.$$

- *Proof*. It is trivial and therefore omitted.
- *A. Stability of Continuous-time Switched Fuzzy Systems*
- First, the novel stability result for systems (1) with $u \equiv 0$ in the fuzzy
- system representation, i.e. the autonomous case, is explored.

III. New Stability Results

- ***Theorem 3.1.*** Suppose there exist a positive definite matrix *P* and
- constants $\lambda_{ij_l} \geq 0,\ i = 1, 2, \ldots, m,\ j_i = 1, 2, \ldots, N_i$ such that
- (9) $$\sum_{i=1}^{N_i} \lambda_{ij_i} (A_{ij_i}^T P + P A_{ij_i}) < 0,\ j_i = 1, 2, \cdots, N_i;$$
- then the system (1) is quadratic stable under the switching law:
- (10) $$\sigma(k) = \arg\min\{\overline{V}_i(x) \overset{\Delta}{=} \max_{j_i}\{x^T (A_{ij_i}^T P + P A_{ij_i}) x < 0,\ j_i = 1, 2, \cdots N_i\}\}.$$
- REMARK. For the sake of outlining how the proofs have been derived, the proof of Theorem 3.1 shall be presented in here; the other proofs are found in the paper.

III. New Stability Results

- *Proof:* From inequality (9) it may well be inferred that
- (11) $$\sum_{i=1}^{N_i} \lambda_{ij_i} x^T (A_{ij_i}^T P + PA_{ij_i})x < 0,\ j_i = 1,2,\cdots N_i$$
- for any $x \neq 0$. Notice that (11) holds true for any $j_i \in \{1,2,\cdots,N_i\}$ and $\lambda_{ij} \geq 0$.
- On the other hand, Lemma 3.1 asserts that there exists at least one i such that
- (12) $$x^T (A_{ij_i}^T P + PA_{ij_i})x < 0$$
- for any j_i. Thus, the switching law (10) is a well-defined one.

- Next, the time derivative of the respective quadratic [8], [10], [15], [19], [26]
- Lyapunov function $V(x(t)) = x^T(t)P\,x(t)$, is to be calculated yielding

$$\frac{d}{dt}V(x(t)) = x^T\left[(\sum_{l=1}^{N_i}\eta_{il}A_{il})^T P + P(\sum_{l=1}^{N_i}\eta_{il}A_{il})\right]x = \sum_{l=1}^{N_i}\eta_{il}x^T\left[A_{il}^T P + PA_{il}\right]x.$$

- It should be noted, here is generated by means of switching law (10).

III. New Stability Results

- By taking (4), (12) into account, one can deduce that

$$dV(x(t))/dt < 0,\ x \neq 0$$

- Hence system (1) is quadratic stable under switching law (10), which ends up this proof.

- Now the stability result for the more important case with , i.e. the non-autonomous case, is presented. It is pointed out that the parallel distributed compensation (PDC) method for fuzzy controller design [23], [24] is used for every fuzzy sub-system. It is shown in the sequel how to design controllers to achieve quadratic stability in the closed loop and under the switching law.

- Note that local fuzzy controller and system (2) have the same fuzzy inference premise variables:

- (13) $R_{ic}^{l}: \ If\ \xi_1\ is\ M_{i1}^{l} \cdots and\ \xi_p\ is\ M_{ip}^{l},\ Then\ u_i(t) = K_{il}x(t),\ l = 1,2,\cdots,N_i, \qquad i = 1,2,\cdots,m.$
- Thus, the overall control is
- (14) $$u_i(t) = \sum_{l=1}^{N_i} \eta_{il} K_{il} x(t)$$

III. New Stability Results

- Therefore the global model of the *i-th* fuzzy sub-system is described by:
- (15) $$\dot{x} = \sum_{l=1}^{N_i} \eta_{il}(t) \sum_{r=1}^{N_i} \eta_{ir} \left(A_{il} + B_{il} K_{ir} \right) x(t).$$
- ***Theorem 3.2.*** Suppose there exist a positive definite matrix *P* and
- constants $\lambda_{ij_l} \geq 0,\ i = 1, 2, ..., m,\ \vartheta_i = 1,2,..., N_i,$ such that
- (16) $$\sum_{i=1}^{N_i} \lambda_{ij_i} \left[(A_{ij_i} + B_{ij_i} K_{i\vartheta_i})^T P + P(A_{ij_i} + B_{ij_i} K_{i\vartheta_i}) \right] < 0,\ j_i, \vartheta_i = 1,2 \cdots, N_i;$$
-
- then, the system (1) along with (13)-(14) is quadratic stable under the following switching law:
- (17) $$\sigma(x) = \arg\min\{\overline{V}_i(x) \overset{\Delta}{=} \max_{j_i, \vartheta_i}\{x^T [(A_{ij_i} + B_{ij_i} K_{i\vartheta_i})^T P + P(A_{ij_i} + B_{ij_i} K_{i\vartheta_i})] x < 0,$$ $$j_i, \vartheta_i = 1,2, \cdots N_i\}\}.$$

III. New Stability Results

- *B. Stability of Discrete-time Switched Fuzzy Systems*
- Again first the case when $u = 0$ in system (5), i.e. the autonomous case, is considered.
- ***Theorem 3.3.*** Suppose there exist a positive definite matrix *P*
- and constants $\lambda_{ij_l} \geq 0,\ i = 1, 2, ..., m,\ \vartheta_i = 1,2,..., N_i,$ such that
- (19) $$\sum_{i=1}^{N_i} \lambda_{ij_i} (A_{ij_i}^T P A_{i\vartheta_i} - P) < 0,\ j_i, \vartheta_i = 1,2,\cdots, N_i,$$
- then the system (5) is quadratic stable under the switching law:
- (20) $$\sigma(k) = \arg\min\{\overline{V}_i(k) \overset{\Delta}{=} \max_{j_i, \vartheta_i}\{x^T (A_{ij_i}^T P A_{i\vartheta_i} - P) x < 0,\ j_i, \vartheta_i = 1,2,\cdots N_i\}\}.$$

III. New Stability Results

The more important case with $u \neq 0$ in system (5), i.e. the non-autonomous case, is considered next. And again the PDC method for fuzzy controller design is used for every fuzzy sub-system.

Namely, it is observed that local fuzzy control and system (6) have the same fuzzy inference premise variables:

(21)
$$R_{ic}^{l}: \; If \;\; \xi_1 \;\; is \;\; M_{i1}^{l} \cdots \; and \;\; \xi_p \;\; is \;\; M_{ip}^{l}, \;\; Then \; u_i(k) = K_{il}x(k),$$
$$l = 1, 2, \cdots, N_i, \;\; i = 1, 2, \cdots, m.$$

Thus, the overall control is

(22)
$$u_i(k) = \sum_{l=1}^{N_i} \eta_{il} K_{il} x(k).$$

Then the global model of the *i-th* sub fuzzy system is described by:

(23)
$$x(k+1) = \sum_{l=1}^{N_i} \eta_{il}(k) \sum_{r=1}^{N_i} \eta_{i\vartheta} \left(A_{il} + B_{il} K_{i\vartheta} \right) x(k).$$

III. New Stability Results

Theorem 3.4 Suppose there exist a positive definite matrix *P* and the set of constants $\lambda_{ij_l} \geq 0,\ i=1,2,...,m,\ \vartheta_i = 1,2,...,N_i,$ such that

(24) $$\sum_{i=1}^{N_i} \lambda_{ij_i} \left[(A_{ij_i} + B_{ij_i} K_{i\vartheta_i})^T P (A_{ip_i} + B_{ip_i} K_{iq_i}) - P \right] < 0,\ \ j_i, \vartheta_i, p_i, q_i = 1,2,\cdots, N_i.$$

Then, the system (5) along with (21)-(22) is quadratic stable under the switching law:

(25) $$\sigma(k) = \arg\min\{\overline{V}_i(k) \overset{\Delta}{=} \max_{j_i,\vartheta_i,p_i,q_i} \{x^T [(A_{ij_i} + B_{ij_i} K_{i\vartheta_i})^T P (A_{ip_i} + B_{ip_i} K_{iq_i}) - P] x < 0,\ j_i, \vartheta_i, p_i, q_i = 1,2,\cdots N_i\}\}.$$

IV. Illustrative Examples and Results

- Results obtained by applying the above developed theory on two examples, one for the continuous-time and one the discrete-time case, and the respective simulations are given below.

- *A. Example 1 for the Continuous-time Case*
- Consider a continuous-time switched fuzzy system, autonomous case, described as follows:

$$R_1^1: \quad If \ x \ is \ M_{11}^1, \ Then \ \dot{x}(t) = A_{11}x(t),$$
$$R_1^2: \quad If \ x \ is \ M_{11}^2, \ Then \ \dot{x}(t) = A_{12}x(t),$$
$$R_2^1: \quad If \ y \ is \ M_{21}^1, \ Then \ \dot{x}(t) = A_{21}x(t),$$
$$R_2^2: \quad If \ y \ is \ M_{21}^2, \ Then \ \dot{x}(t) = A_{22}x(t),$$

- where the system matrices are

IV. Illustrative Examples and Results

$$A_{11} = \begin{bmatrix} -17 & -0.0567 \\ 4.93 & -0.983 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} -10 & 0.216 \\ -0.0132 & -45.29 \end{bmatrix};$$

$$A_{21} = \begin{bmatrix} -50 & -0.042 \\ 0.008 & -10 \end{bmatrix}, \qquad A_{22} = \begin{bmatrix} -40 & -0.0867 \\ 0.047 & -120 \end{bmatrix}.$$

- The fuzzy sets $M_{11}^1, M_{11}^2, M_{21}^1, M_{21}^2$ are represented, respectively,
- by means of the following membership functions:

$$\mu_{11}^1(x) = 1 - \frac{1}{1 + e^{-2x}}, \quad \mu_{11}^2(x) = \frac{1}{1 + e^{-2x}}$$

$$\mu_{21}^1(y) = 1 - \frac{1}{1 + e^{(-2(y-0.3))}}, \quad \mu_{21}^2(y) = \frac{1}{1 + e^{(-2(y-0.3))}}$$

IV. Illustrative Examples and Results

- For

$$\sum_{i=1}^{2} \lambda_{ij_i} (A_{ij_i}^T P + P A_{ij_i}) < 0, \quad j_i = 1, 2,$$

-
- by choosing $\lambda_{ij_i} = 1$, one can find the following *P* matrix

$$P = \begin{bmatrix} 0.0159 & 0.0001 \\ 0.0001 & 0.0071 \end{bmatrix}$$

- Then system is quadratic asymptotically stable under the following switching law

$$\sigma(x) = \arg\min\{\overline{V}_i(x) \stackrel{\Delta}{=} \max_{j_i}\{x^T (A_{ij_i}^T P + P A_{ij_i}) x < 0, \ j_i = 1,2\}\}$$

- Figure 2 below depicts the obtained simulation results for the controlled evolution of system state variables with the initial state vector $x(0) = [2 \quad 2]^T$, chosen at random.

IV. Illustrative Examples and Results

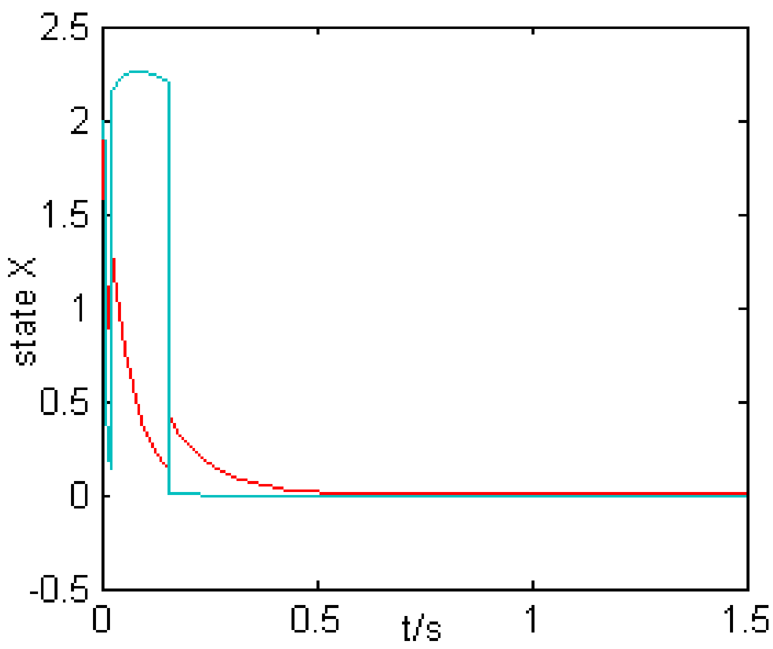


Figure 2. Response evolution of system state variables $x_1(t),\ x_2(t)$

IV. Illustrative Examples and Results

- *B. Example 2 for the Discrete-time Case*

- Let consider a discrete switched fuzzy system, a non-autonomous case, described as follows:

$$R_1^1 : If \;\; x(k) \;\; is \;\; M_{11}^1, \; Then \; x(k+1) = A_{11}x(k) + B_{11}u(k),$$
$$R_1^2 : If \;\; x(k) \;\; is \;\; M_{11}^2, \; Then \; x(k+1) = A_{12}x(k) + B_{12}u(k),$$
$$R_2^1 : If \;\; y(k) \;\; is \;\; M_{21}^1, Then \; x(k+1) = A_{21}x(k) + B_{21}u(k),$$
$$R_2^2 : If \;\; y(k) \;\; is \;\; M_{21}^2, Then \; x(k+1) = A_{22}x(k) + B_{22}u(k),$$

where system matrices are

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -0.0493 & -1.0493 \end{bmatrix}, \qquad B_{11} = \begin{bmatrix} 0 \\ 0.4926 \end{bmatrix};$$

IV. Illustrative Examples and Results

$$A_{12} = \begin{bmatrix} 0 & 1 \\ -0.0132 & -0.4529 \end{bmatrix}, \qquad B_{12} = \begin{bmatrix} 0 \\ 0.1316 \end{bmatrix};$$

$$A_{21} = \begin{bmatrix} 0 & 1 \\ -0.2 & -0.1 \end{bmatrix} \quad B_{21} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A_{22} = \begin{bmatrix} 0.2 & 1 \\ -0.8 & -0.9 \end{bmatrix} \quad B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The fuzzy sets $M_{11}^1, M_{11}^2, M_{21}^1, M_{21}^2$ are represented, respectively, by means of the following membership functions:

$$\mu_{11}^1(x(k)) = 1 - \frac{1}{1+e^{-2x(k)}}, \ \mu_{11}^2(x(k)) = \frac{1}{1+e^{-2x(k)}}$$

$$\mu_{21}^1(y(k)) = 1 - \frac{1}{1+e^{(-2(y(k)-0.3))}}, \ \mu_{21}^2(y(k)) = \frac{1}{1+e^{(-2(y(k)-0.3))}}.$$

IV. Illustrative Examples and Results

The state feedback gains of subsystems are obtained as

$$K_{11} = [-0.131 \quad -0.1148],\ K_{12} = [-0.0623 \quad -2.302],$$

$$K_{21} = [1.8 \quad 1.9],\ K_{22} = [-0.7 \quad 1.3]$$

For

$$\sum_{i=1}^{2} \lambda_{ij_i} \left[(A_{ij_i} + B_{ij_i} K_{i\vartheta_i})^T P (A_{ip_i} + B_{ij_i} K_{iq_i}) - P \right] < 0,\ j_i, \vartheta_i, p_i, q_i = 1, 2,$$

by choosing $\lambda_{ij_i} = 1$, one can obtain the P matrix

$$P = \begin{bmatrix} 0.3563 & -0.0087 \\ -0.0087 & 0.1780 \end{bmatrix}.$$

IV. Illustrative Examples and Results

Then, the system is quadratic stable under the following switching law

$$\sigma(k) = \arg\min\{\overline{V}_i(k) \overset{\Delta}{=} \max_{j_i,\vartheta_i,p_i,q_i} \{x^T[(A_{ij_i} + B_{ij_i}K_{i\vartheta_i})^T P(A_{ip_i} + B_{ip_i}K_{iq_i}) - P]x < 0,\ j_i, \vartheta_i, p_i, q_i = 1,2\}\}.$$

- Figure 3 below depicts the obtained simulation results for the controlled evolution of system state variables with the initial state vector $x(0) = [5 \quad 1]^T$, chosen at random, and the sampling period T_s = 0.05 s.

IV. Illustrative Examples and Results

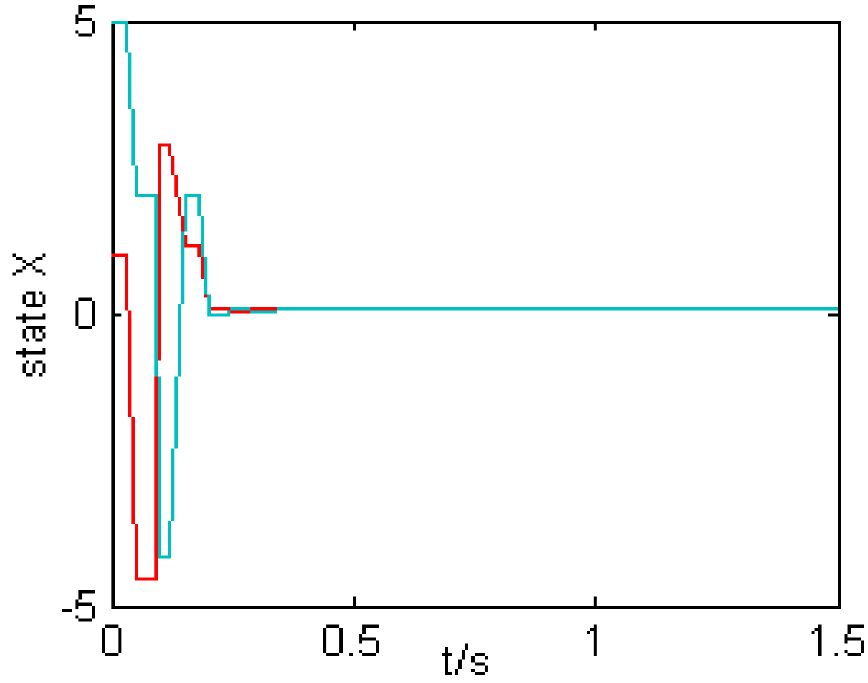


Figure 3. Response evolution of system state variables $x_1(k),\ x_2(k)$

A New Potential for Intelligent Control: Synergy of Switched and Fuzzy Systems

V. Conclusions

- An innovated representation model for the class of switched fuzzy systems, both continuous and discrete-time, is proposed.
- For both cases, new sufficient conditions for quadratic asymptotic stability of the control system with the given switching laws (Theorems 3.1 thru 3.4) have been derived via the common Lyapunov function approach. Following these new stability results, only the stability of a certain combination of subsystem matrices has to be checked, which is considerably easier to carry out.
- Lastly but not least, appropriate stabilizing switching laws in the state-variable dependent form have been synthesized for both these cases of fuzzy switched systems. Simulation results demonstrate that a quality control performance that can be achieved in the closed loop system.

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- Switched Fuzzy Systems: Representation Modelling, Stability, and Control Design, ***Studies in Computational Intelligence Volume 109*** – *Intelligent Techniques and Tools for Novel Systems Architectures*, edited by P. Chountas, I. Petrounias and J. Kacprzyk (Editor-in-Chief). Springer-Verlag, Berlin Heidelberg, 2008, pp. 169-184.

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