3D Surface Reconstruction

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Outline

The surface reconstruction problem

Computational Intelligence approaches

The regression problem

Neural-based techniques for regression

Why a hierarchical approach?

Radial Basis Function Neural network

Hierarchical RBF model

Support Vector Machine for Regression (SVR)

The Hierarchical SVR model

Conclusions

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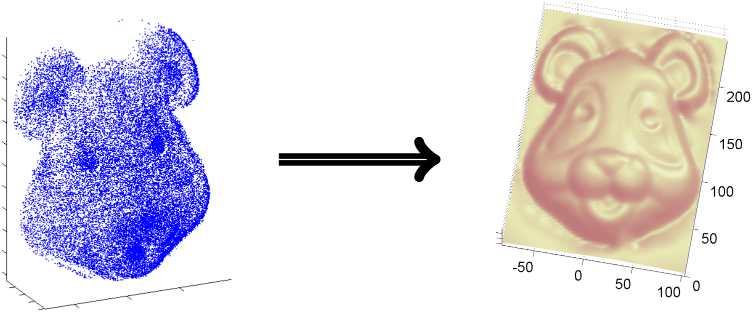

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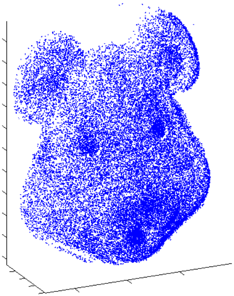

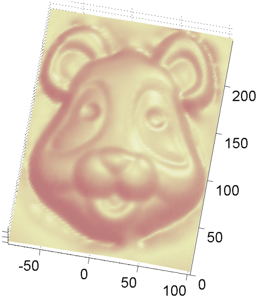

- In computational intelligence field:
  - surface reconstruction $\rightarrow$ function approximation / regression
  - point cloud $\rightarrow$ examples
  - surface $\rightarrow$ generalization

3D Models

- 3D models are used in many applications

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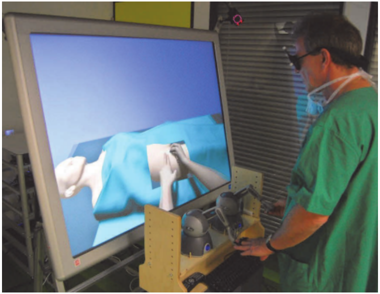

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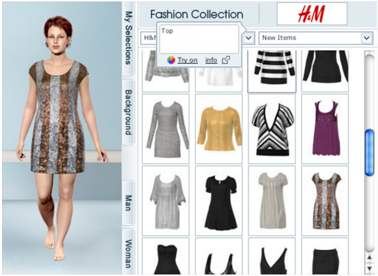

3D Models

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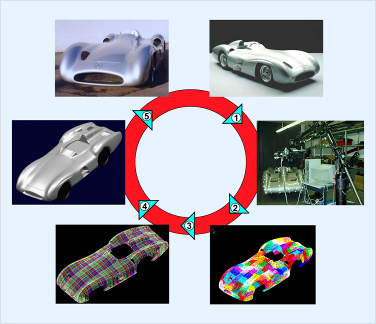
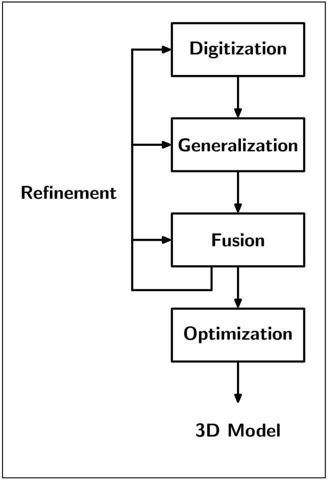
  - Virtual fashion
  - Reverse engineering
  - ...



3D reconstruction pipeline



Several features characterize the reconstruction problem:

- the inherent structure of the data (e.g., contour) can be easily incorporated in the model;
- the topology of the surface should be known a-priori or can be obtained from enough dense data;
- the class of the object: model-based reconstruction.

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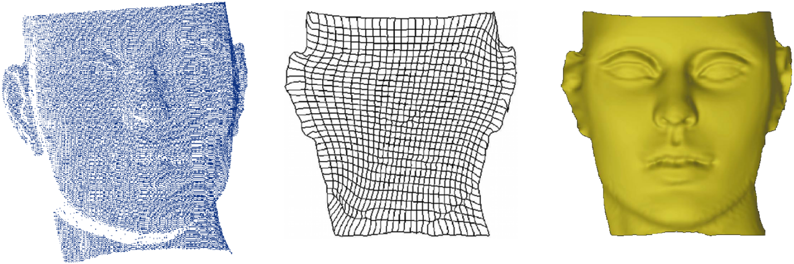
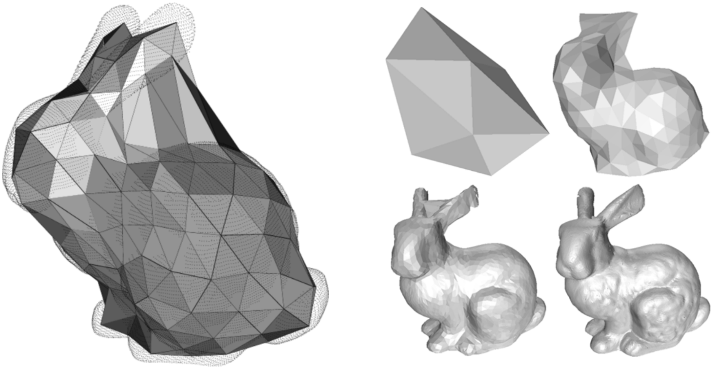
Self-Organizing Maps



- SOMs provide a smooth mapping of similar examples.
- The topological structure can be used as mesh for surface representation.
- Particular attention has to be paid to the problem of distributing the SOM units on the boundary of the training set.

Self-Organizing Maps (2)



- Dense SOM can generate unrealistic meshes.
- Pruning and iterative adaptation provide better results:
  - removing of unstable vertices;
  - subdivision of large triangles and refinement.

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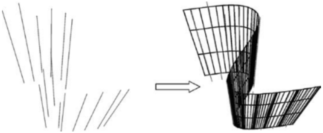
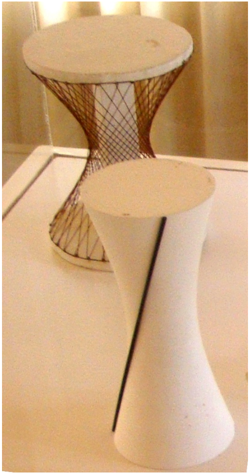
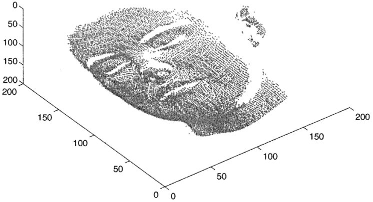
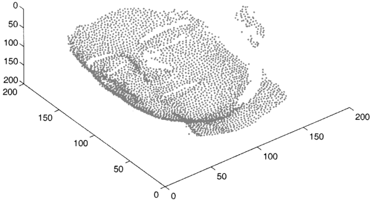




- Ruled surfaces are commonly used in CAD.
- Each 3D points line is transformed in a 6-dimensional point that describes the line;
- in that 6D space, the SOM is trained;
- the backward transformation allows to obtain the ordered set of lines that defines the surface.

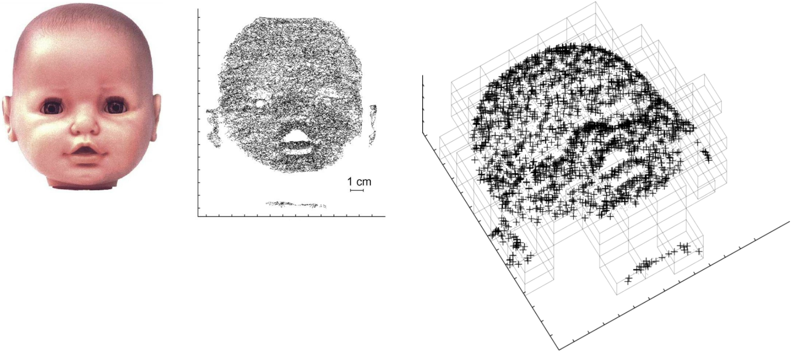
FOSART





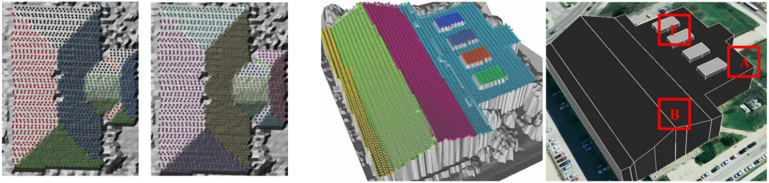
- Clustering provides filtering through local averaging.
- Topology preserving algorithms limits the averaging only on similar points.
- Fully self-organizing simplified adaptive resonance theory (FOSART) has been used for filtering a dense 3D point cloud affected by noise.

Enhanced Vector Quantization



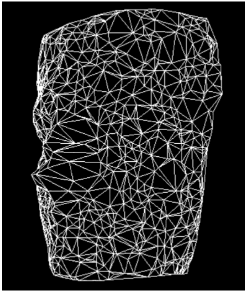
- Extension of the Neural-Gas model:
  - optimized for low dimensionality spaces;
  - linear scaling and parallelizable.

Fuzzy k-means



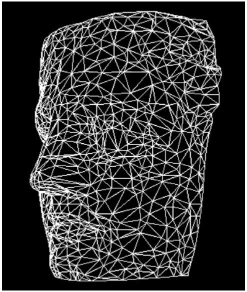
- Use of computational intelligence techniques as intermediate step.
- Segmentation of normals of the surfaces points through fuzzy k-means clustering;
- description of the surface as a collection of planar patches.

Evolutionary



- Evolutionary optimization for triangle mesh.

Evolutionary



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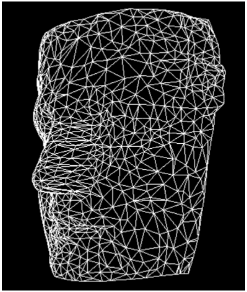

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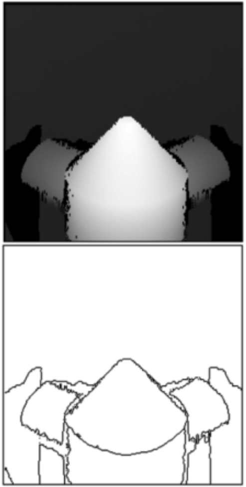

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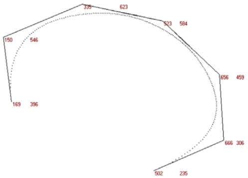

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- Artificial immune system for B-spline knot positioning.

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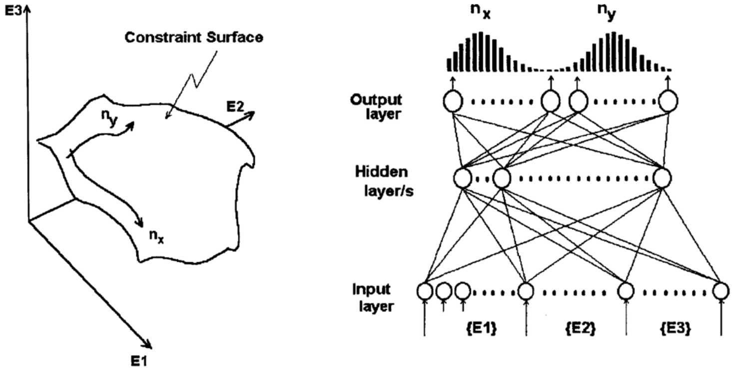
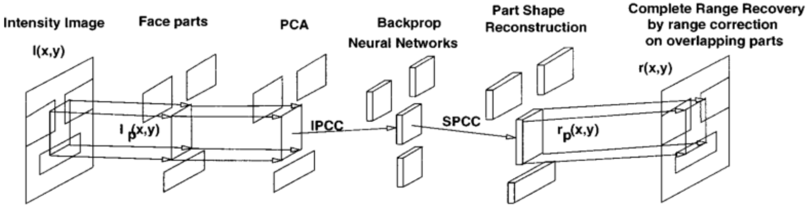
Neural networks based 3D recovery



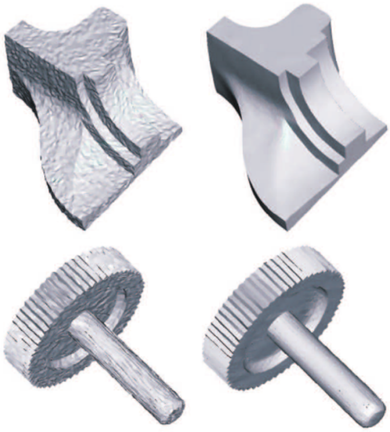
- A feed-forward neural network is fed with the pixel intensities of three images to predict the normal of the depicted surface.
- From the normals, the surface can be obtained.

Neural networks based 3D recovery (2)



- A face image is segmented in face parts.
- A neural network is trained to learn the 3D shape PCA weights from the intensity PCA weights.
- The 3D shape of the parts is then blended to obtain the 3D shape of the face.

Fuzzy smoothing



- Post-reconstruction processing.
- Fuzzy filtering of patches normals.

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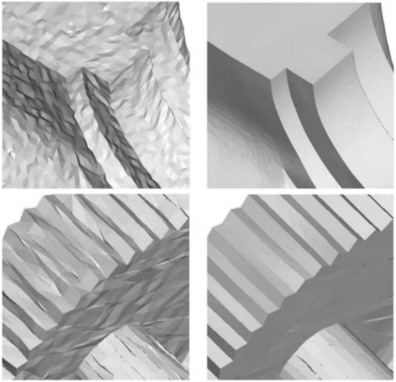

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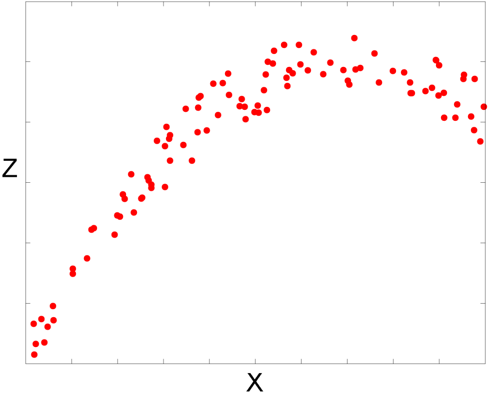

- $S = \{(x_1, z_1), \ldots, (x_n, z_n)\}$, $x_i \in \mathbb{R}^D$, $1 \leq i \leq n$
  - data affected by noise

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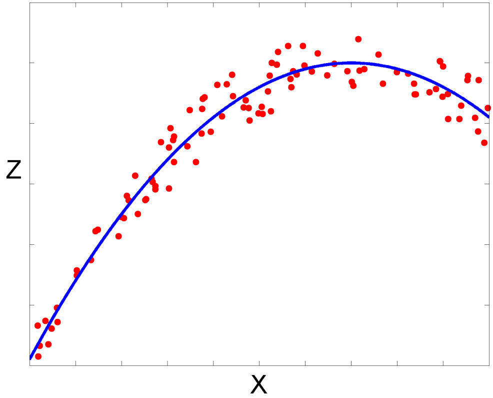

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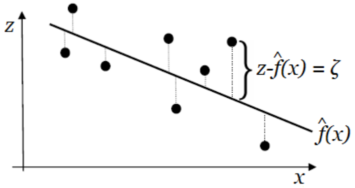
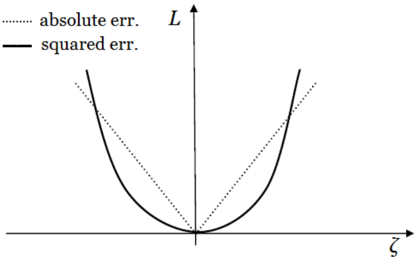
- $\hat{f} : \mathbb{R}^D \to \mathbb{R}$ such that $\hat{f}(x) \approx z = f(x) + \epsilon$ where ϵ is a r.v. with zero mean and finite standard deviation

The Regression Problem (2)

- The solution $\hat{f}$ has to be chosen in order to minimize a given training error function, L:
  - $L = \sum_{i=1}^{n} E(\hat{f}(x_i), z_i)$





Solution by means of basis functions

- approximation as combination of basis functions

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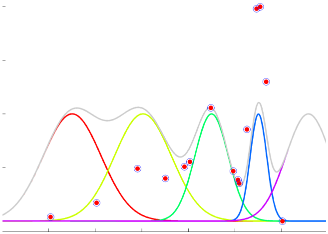

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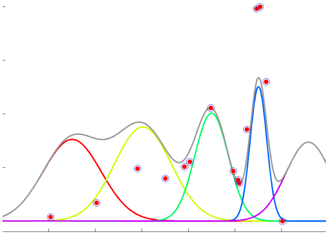

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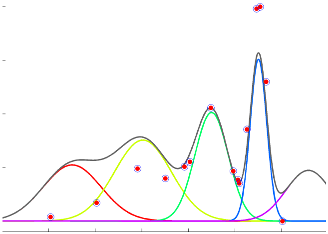

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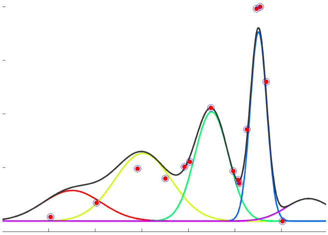

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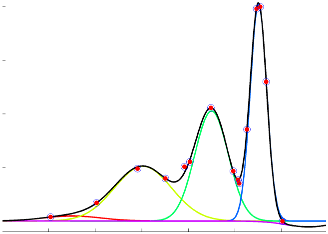

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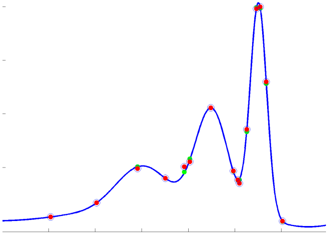
- distance between target function and approximation



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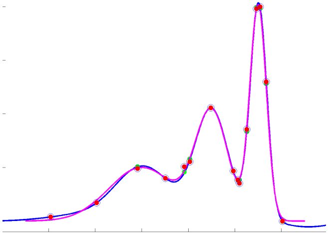

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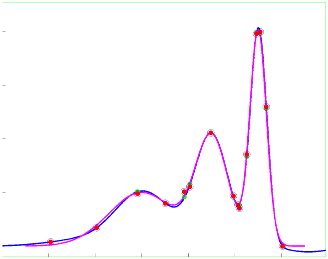

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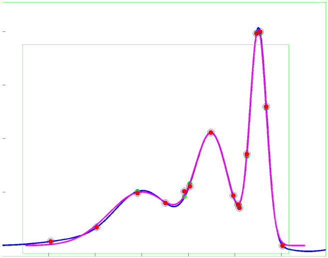

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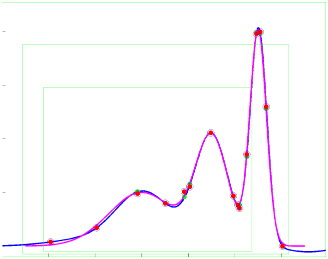

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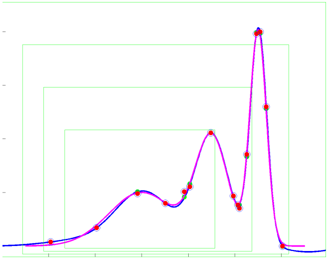

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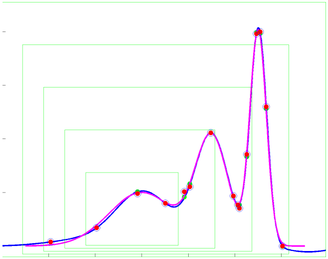

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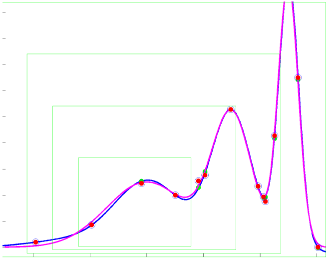

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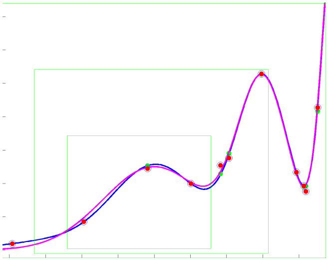

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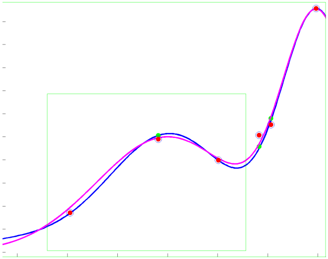

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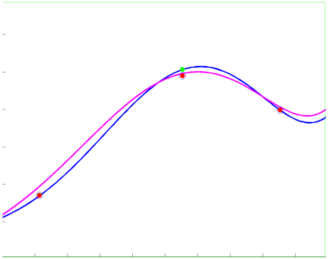

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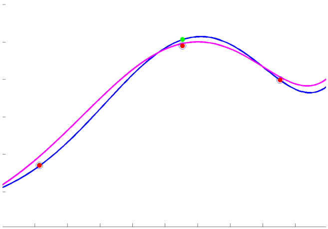

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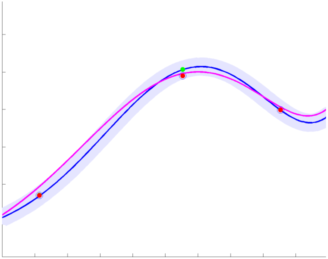

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Neural-based techniques for regression

- Generalization ability
  - Noise filtering effect
- Non-linear model
- Small number of hyperparameters
- Good trade-off between accuracy and computational efficiency
- Smooth solutions
- Online learning

Hierarchical approaches

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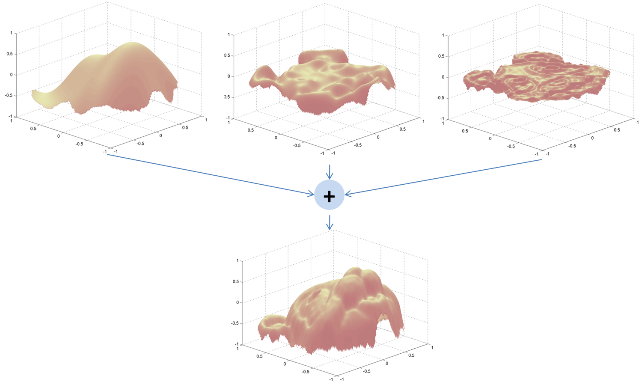
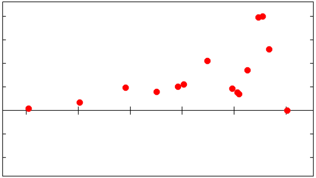


The Hierarchical Model



input data

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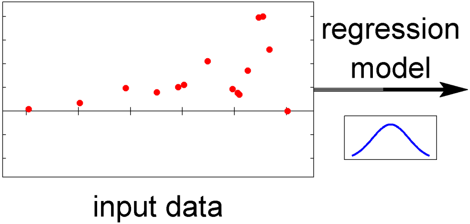

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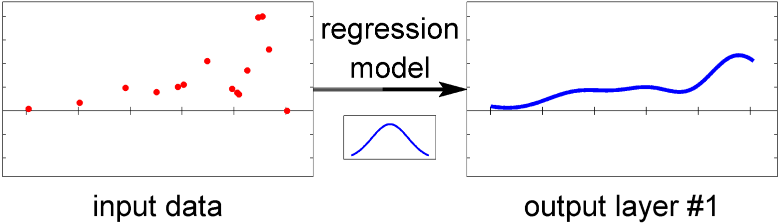
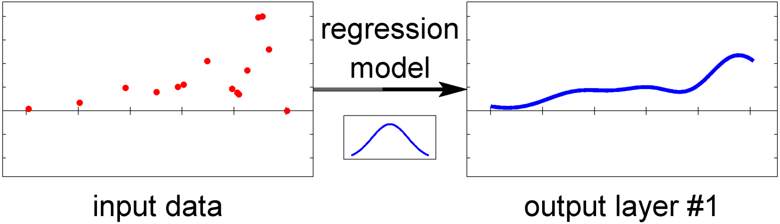
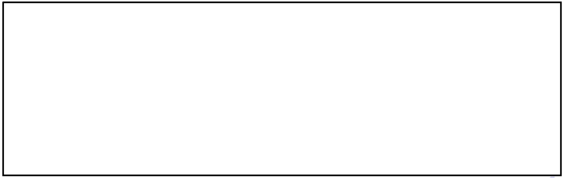


The Hierarchical Model





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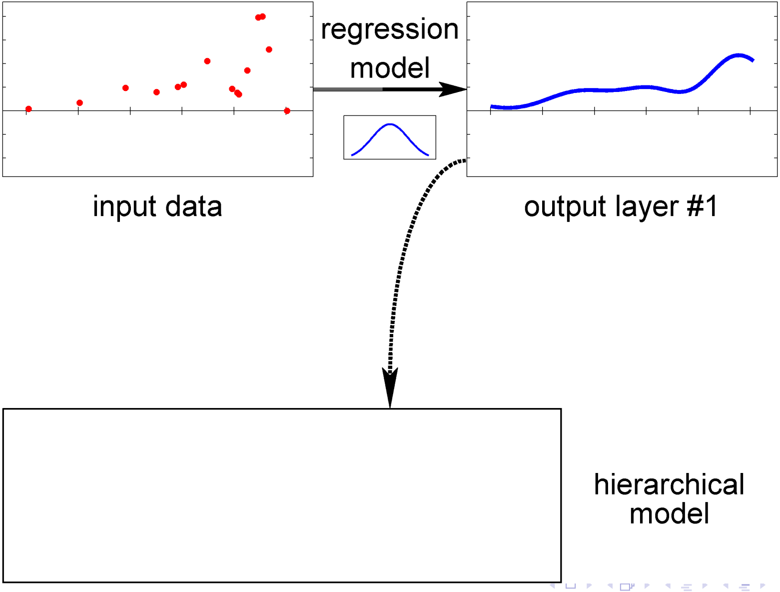

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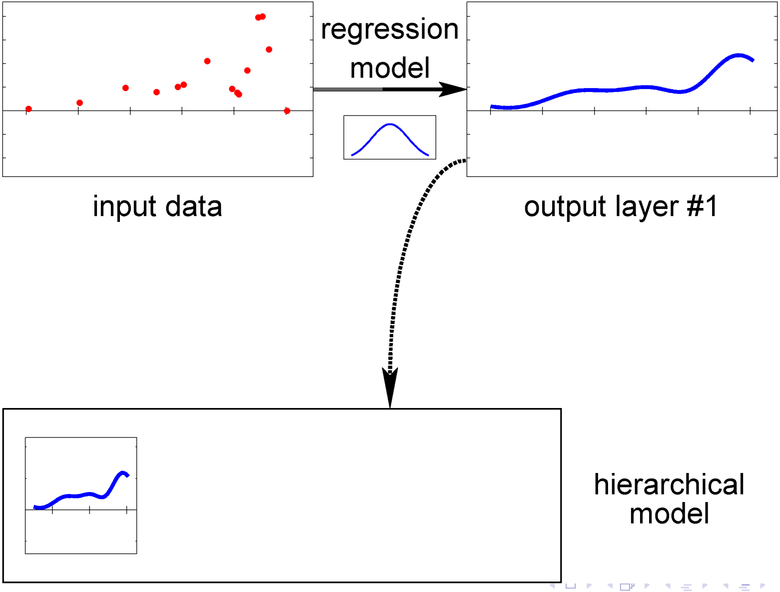

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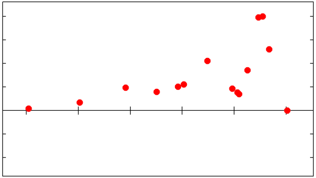

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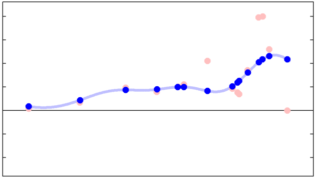

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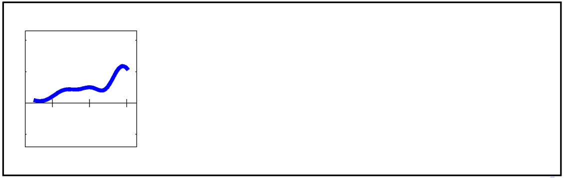
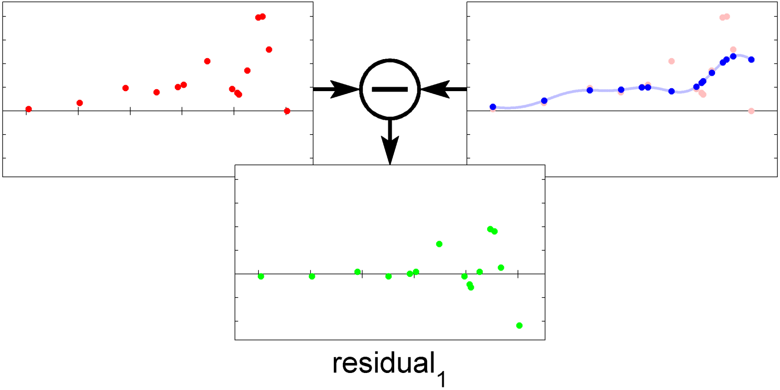
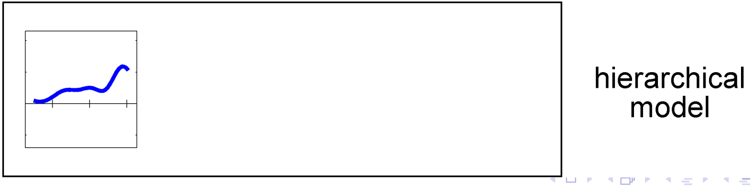


hierarchical model

The Hierarchical Model





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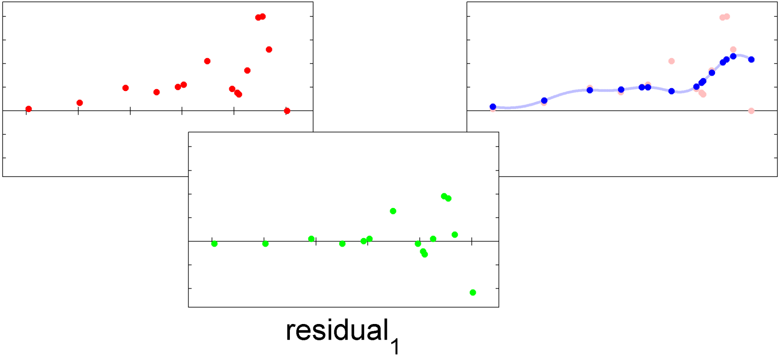

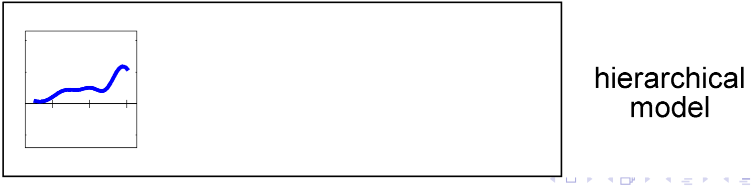

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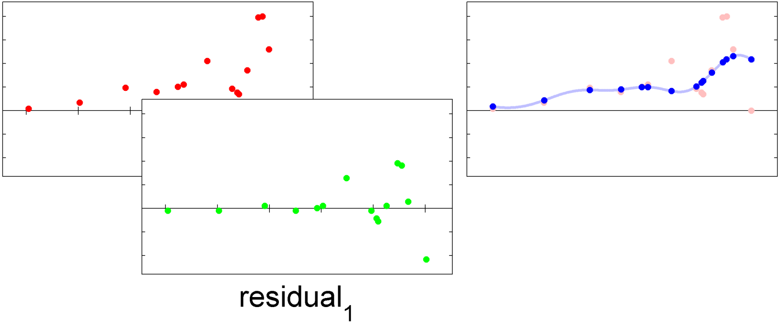

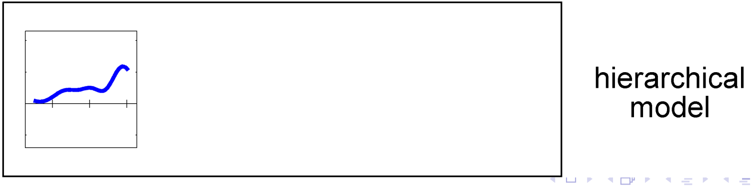

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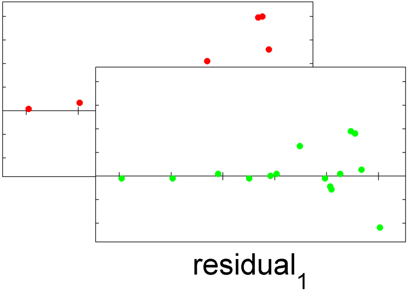
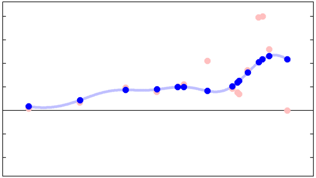
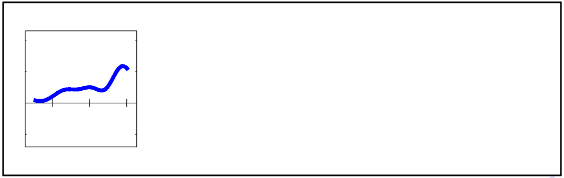
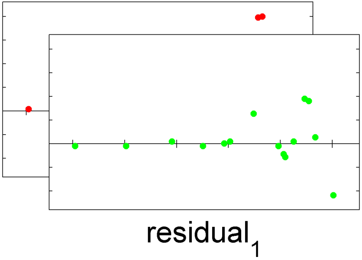


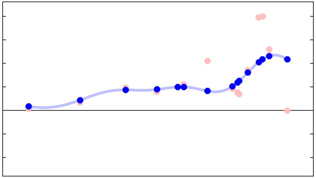


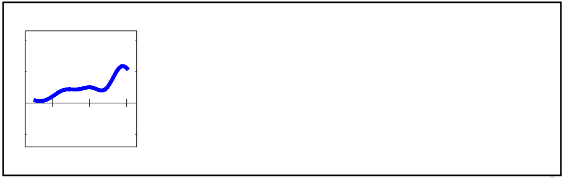


hierarchical
model

The Hierarchical Model

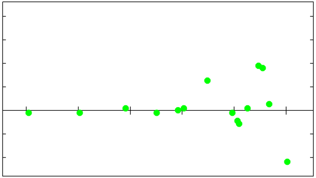




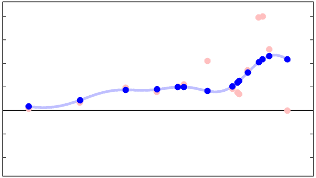


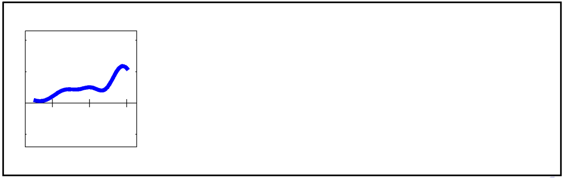
hierarchical model

The Hierarchical Model



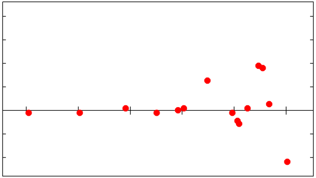
residual$_1$



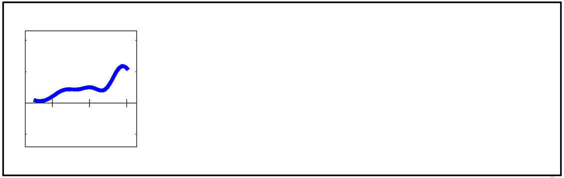


hierarchical model

The Hierarchical Model

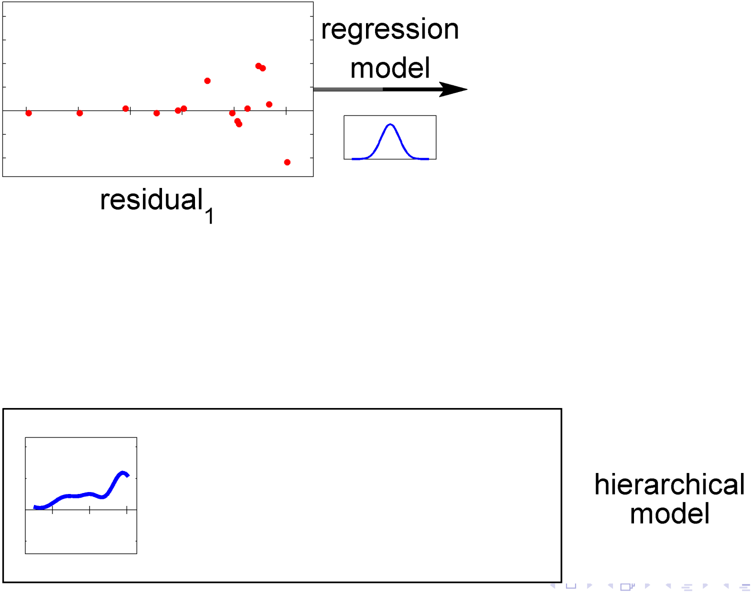


residual$_1$



hierarchical model

The Hierarchical Model



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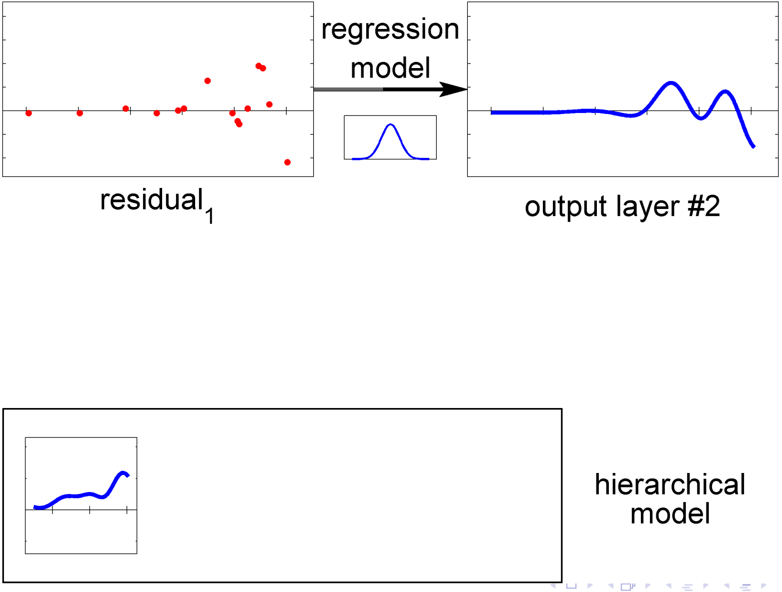

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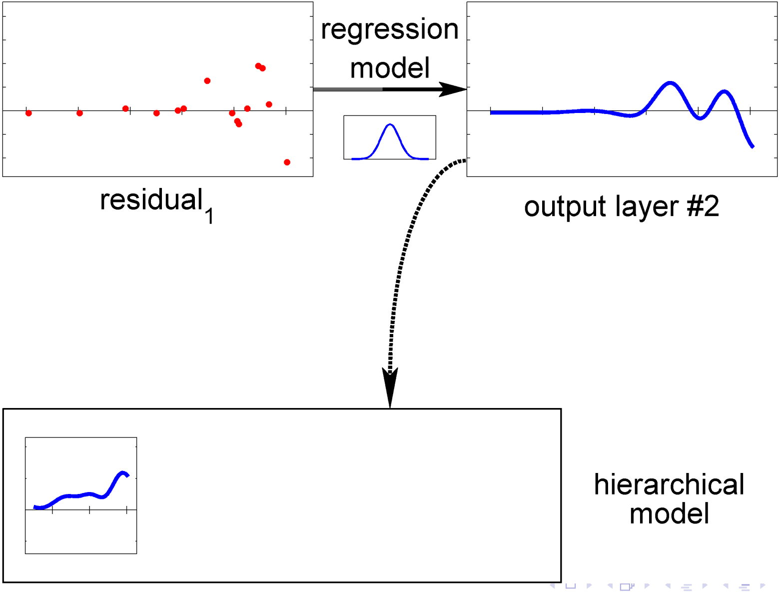

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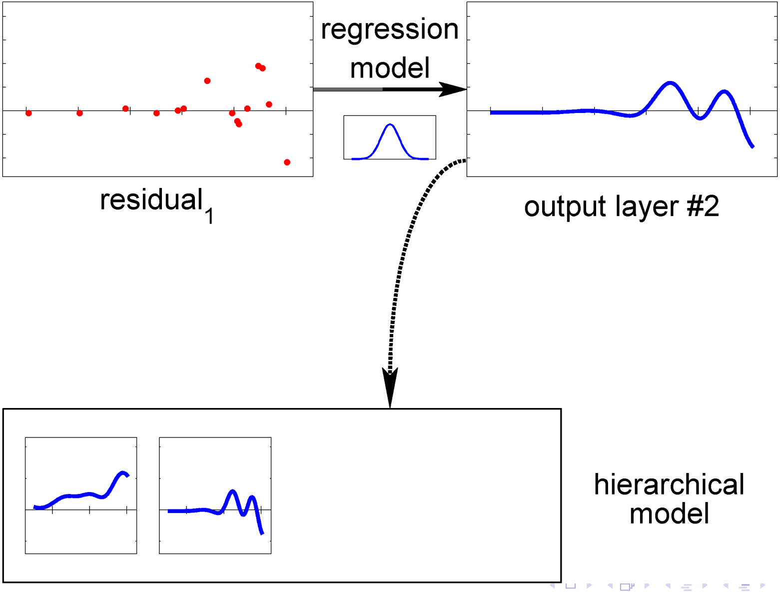

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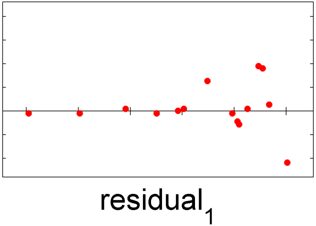

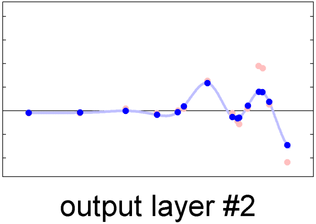

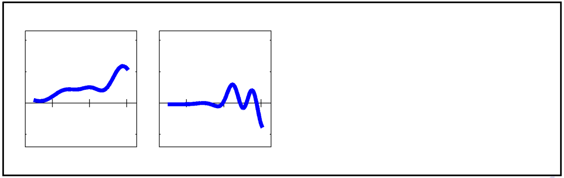

hierarchical model

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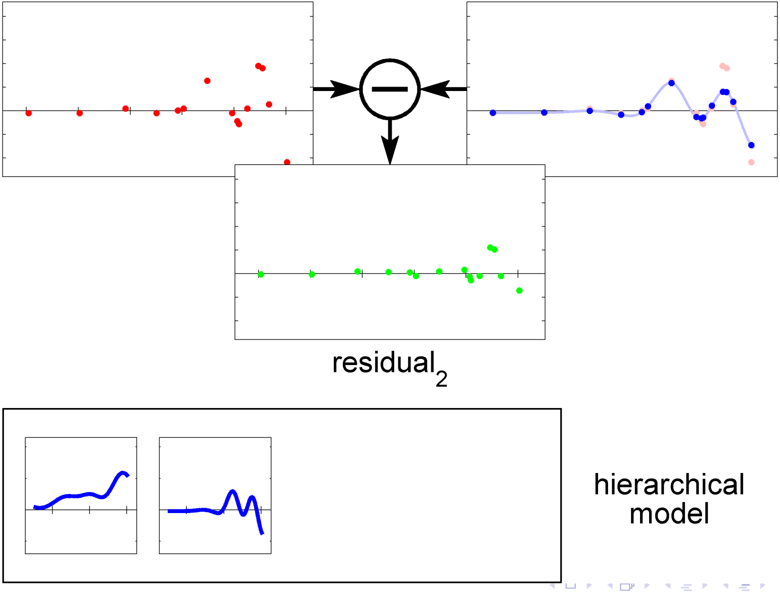

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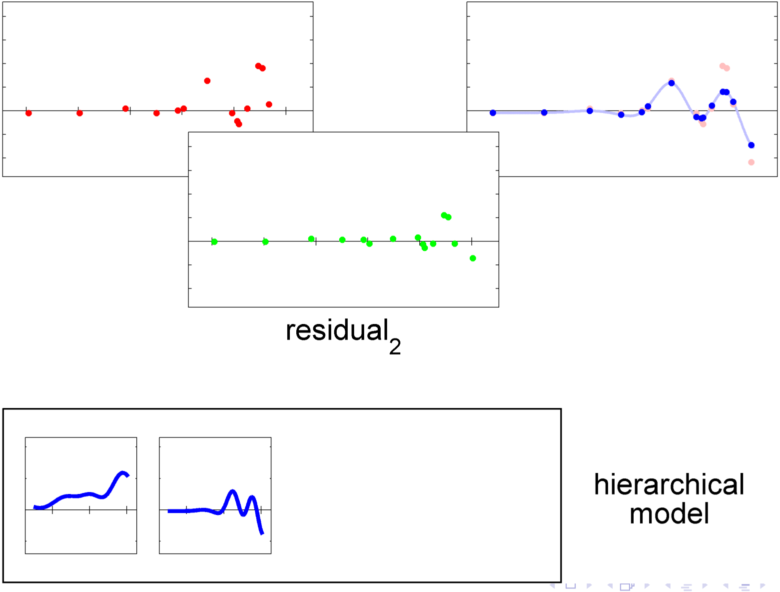

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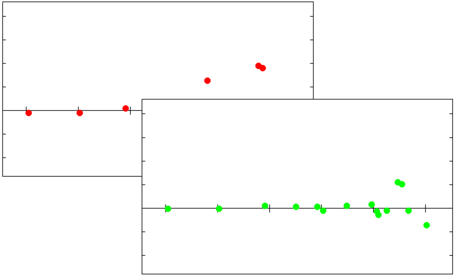
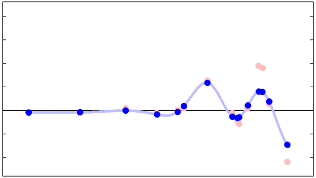
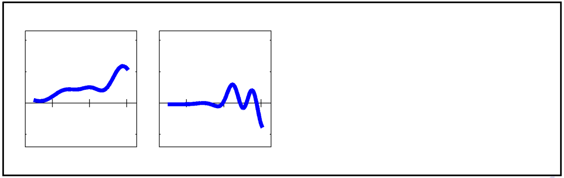


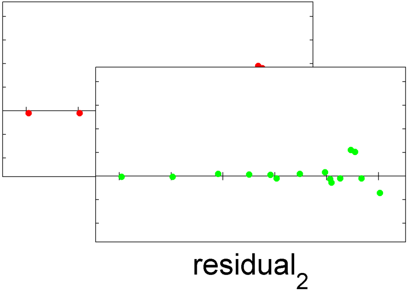
residual$_2$

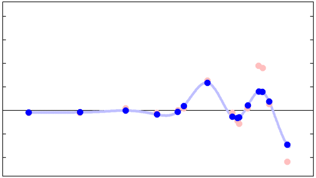


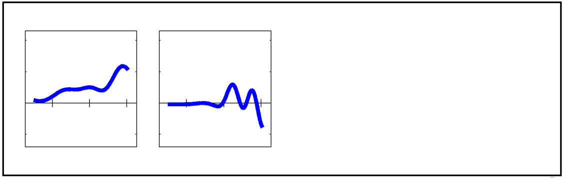


hierarchical model

The Hierarchical Model







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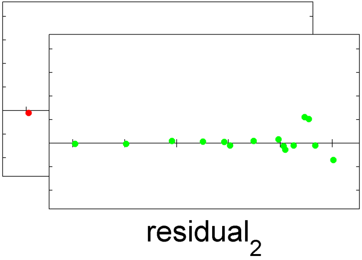
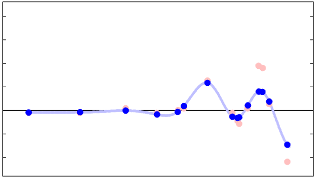
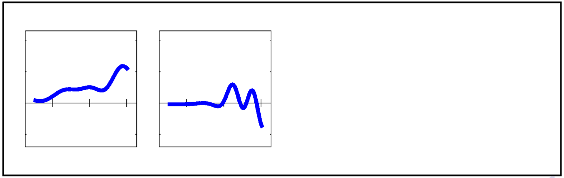
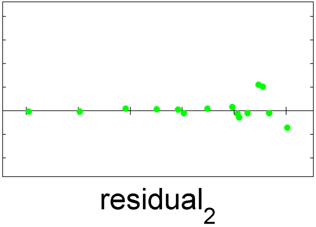


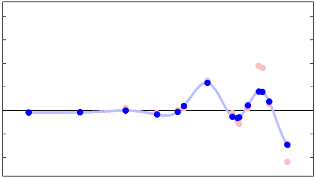


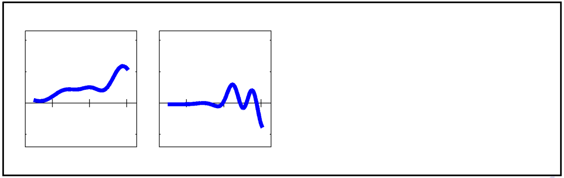


hierarchical model

The Hierarchical Model

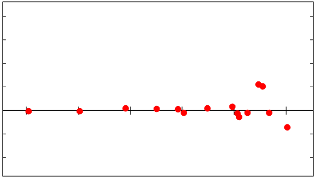




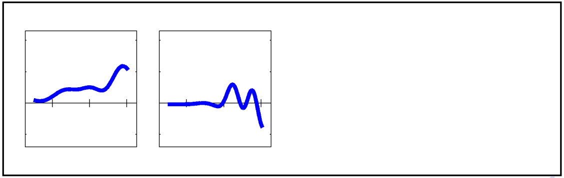


hierarchical model

The Hierarchical Model

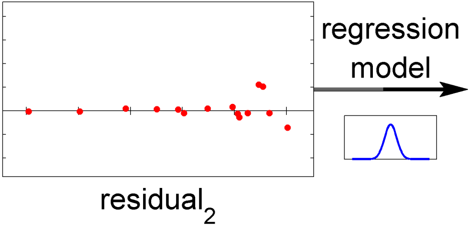


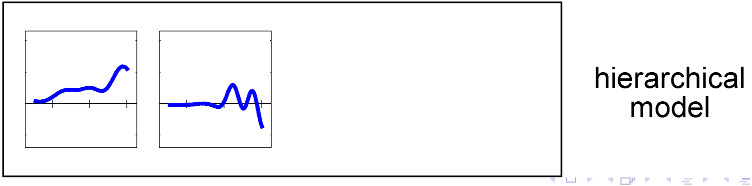
residual$_2$



hierarchical
model

The Hierarchical Model





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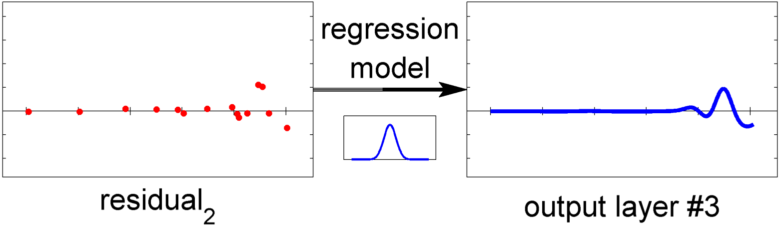
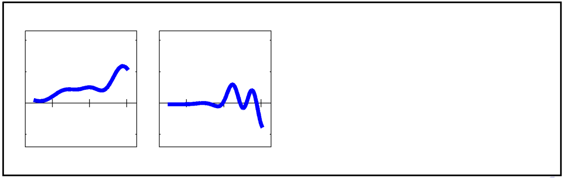
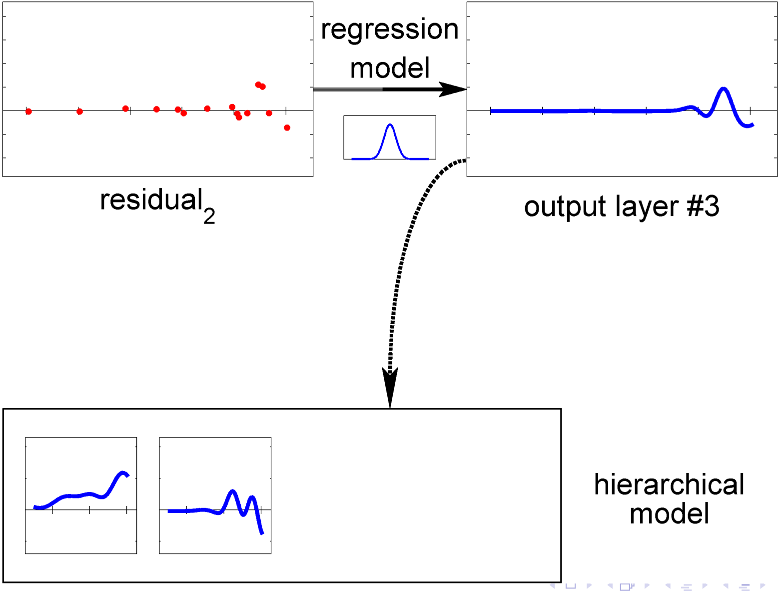




hierarchical
model

The Hierarchical Model



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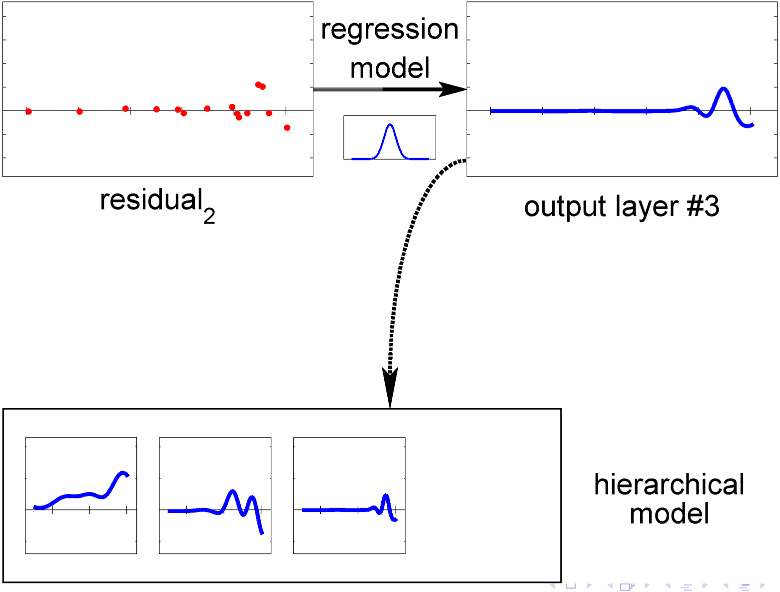

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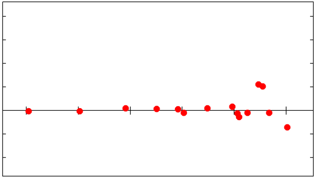
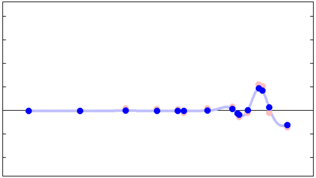
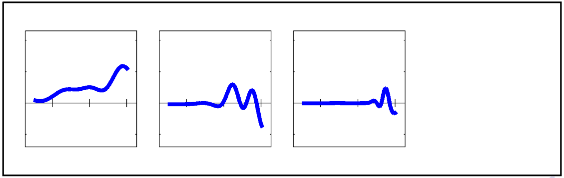


residual$_2$

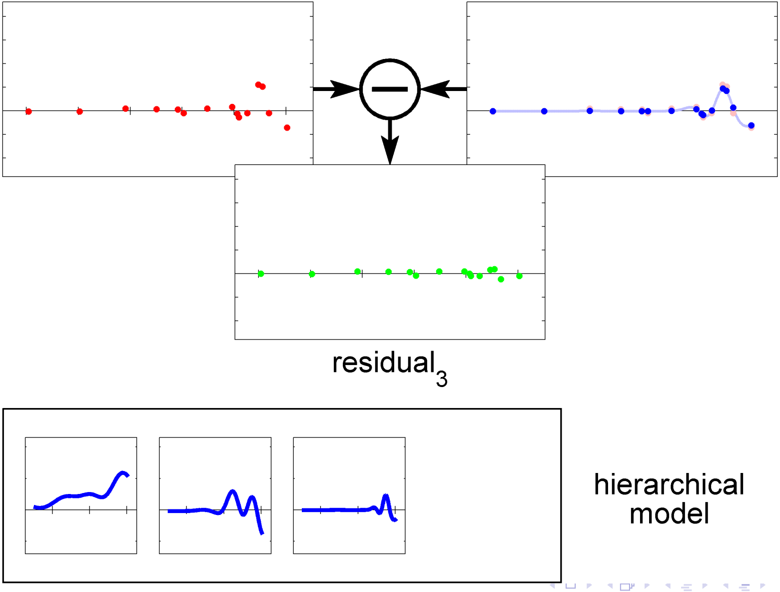


output layer #3



hierarchical model

The Hierarchical Model



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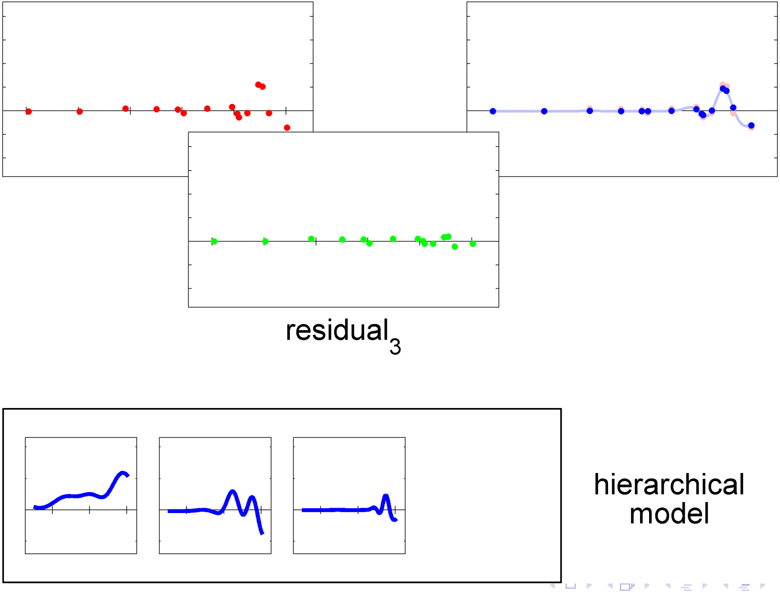

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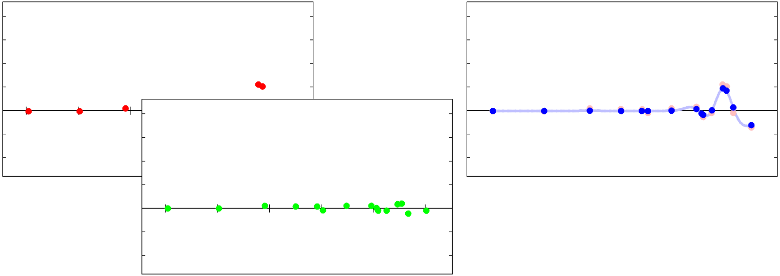

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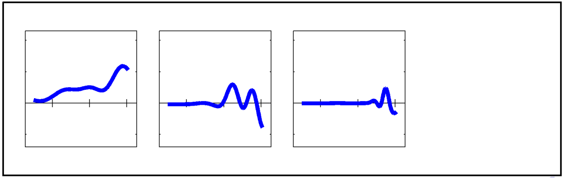
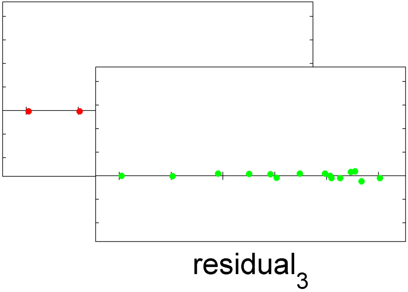
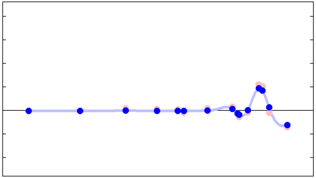
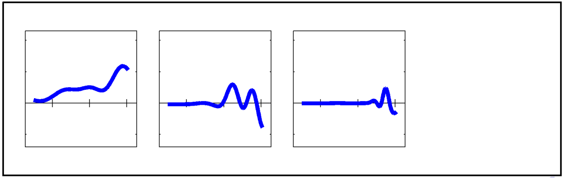


hierarchical
model

The Hierarchical Model

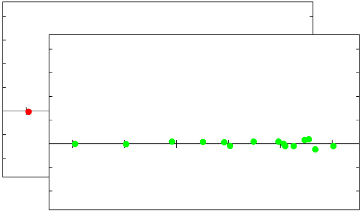




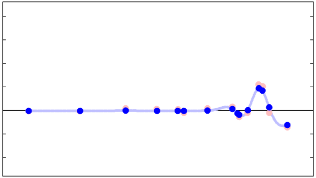


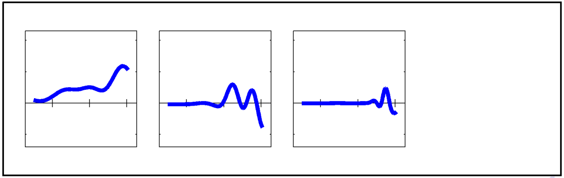
hierarchical model

The Hierarchical Model



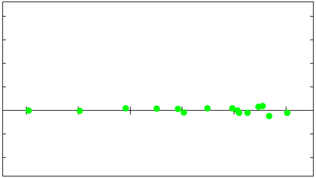
residual$_3$



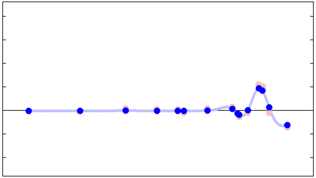


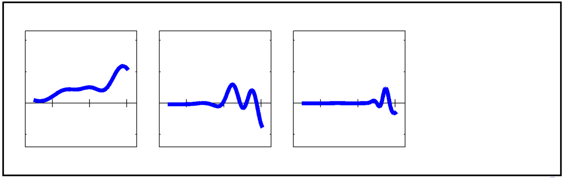
hierarchical model

The Hierarchical Model



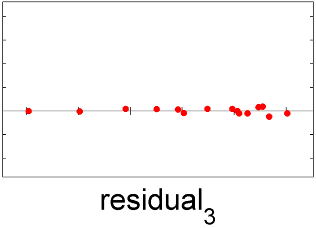
residual$_3$

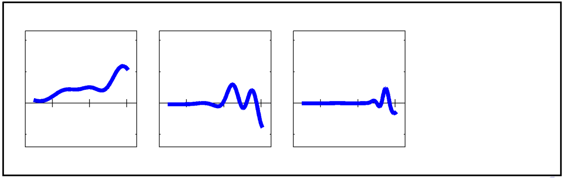




hierarchical model

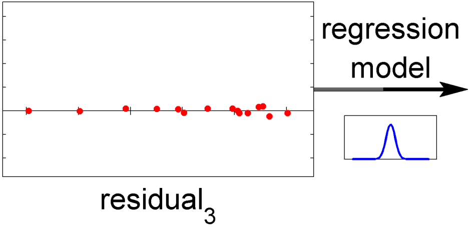
The Hierarchical Model

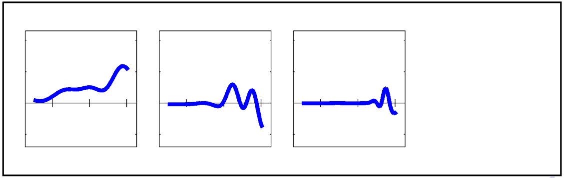




hierarchical
model

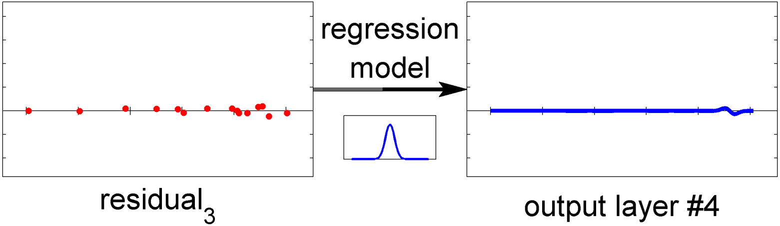
The Hierarchical Model

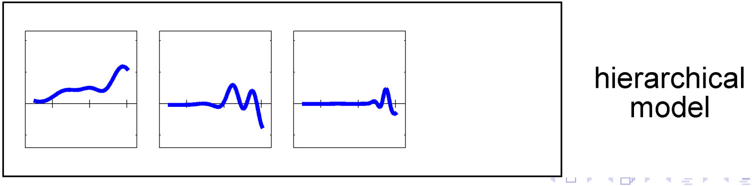




hierarchical model

The Hierarchical Model





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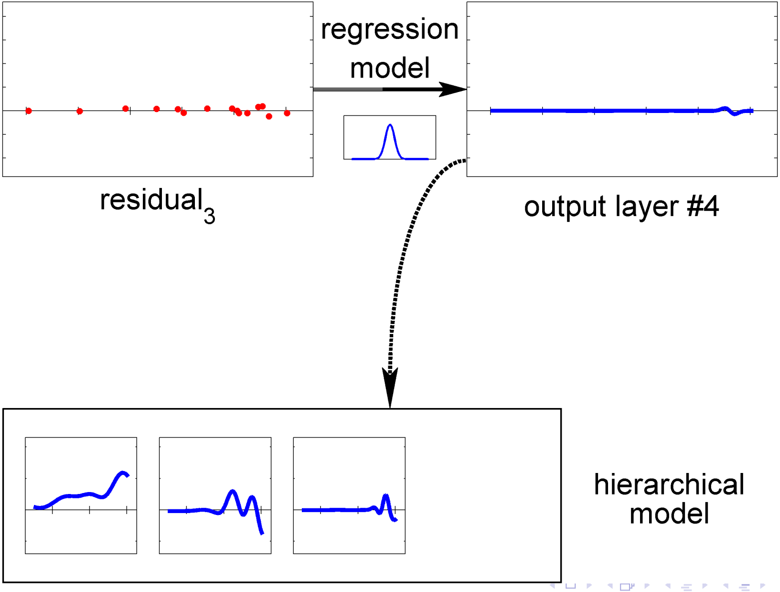

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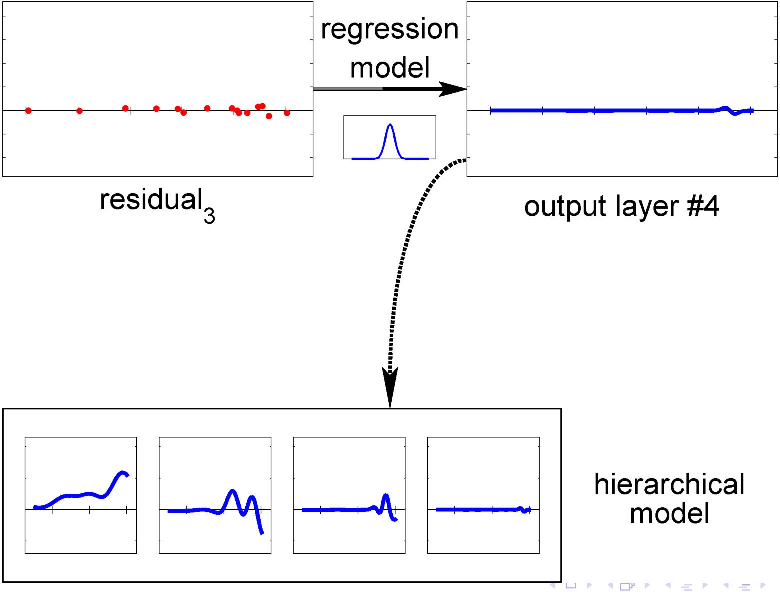

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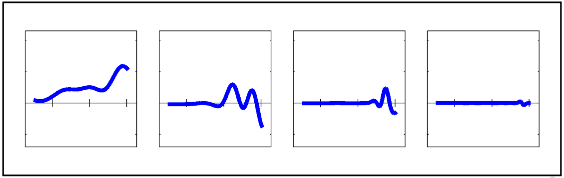

hierarchical
model

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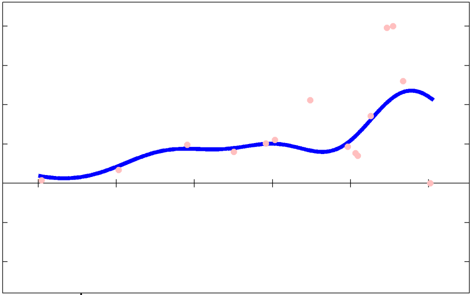
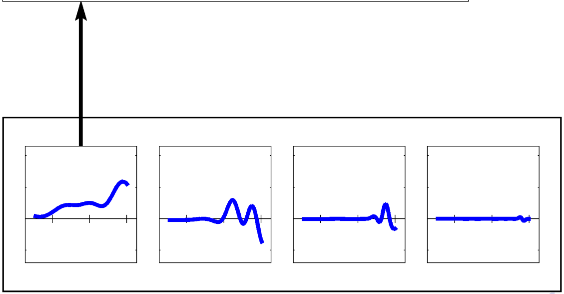


hierarchical model output up to the first layer



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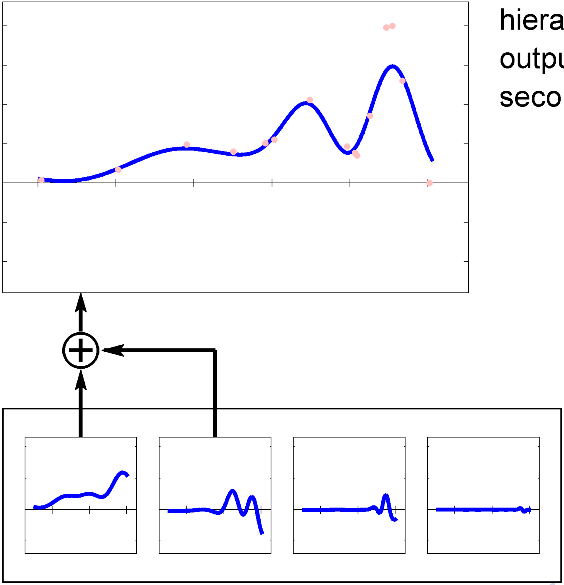

hierarchical model output up to the second layer

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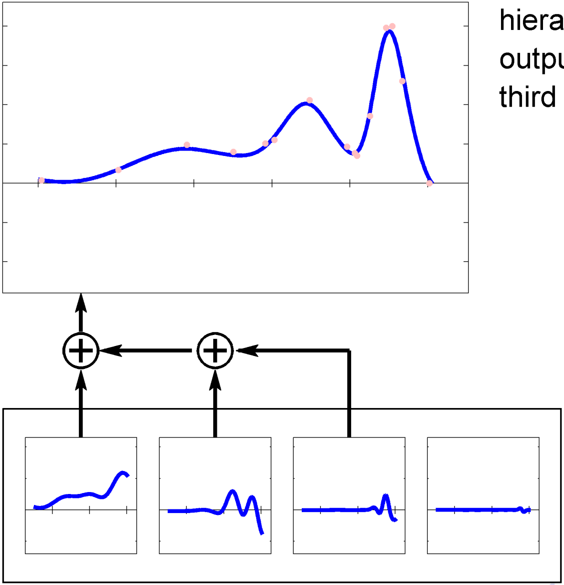

hierarchical model output up to the third layer

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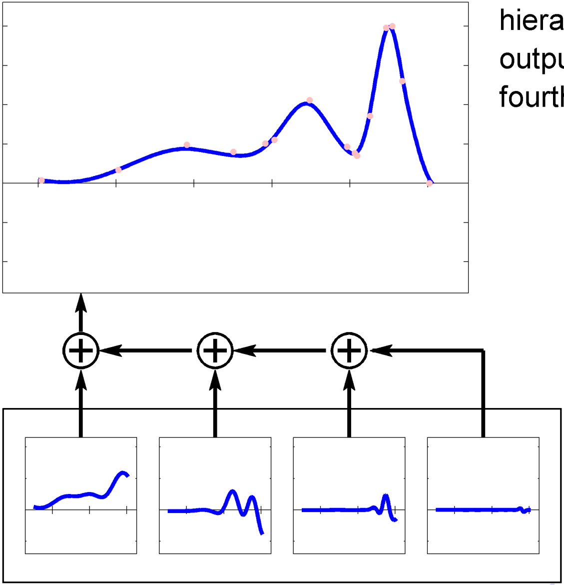
The Hierarchical Model



hierarchical model output up to the fourth layer

hierarchical model

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- Face the situations where the standard models are not able to compute an accurate solution

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Hierarchical neural-based models

- RBF
  - Hierarchical RBF
    - Local approach: fast configuration

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- RBF
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    - Local approach: fast configuration
- SVM
  - Hierarchical SVM
    - Global approach: computational complexity regardless of the number of input variables

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- The output of the model is computed as linear combination of radial basis functions
- If the RBFs (or units) are normalized spherical Gaussians the output has the form:

$$f(x) = \sum_{k=1}^{M} w_k \frac{1}{\sigma_k \sqrt{2\pi}} e^{\left(-\frac{||x-\mu_k||^2}{2\sigma_k^2}\right)}$$

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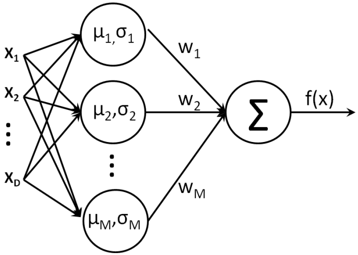

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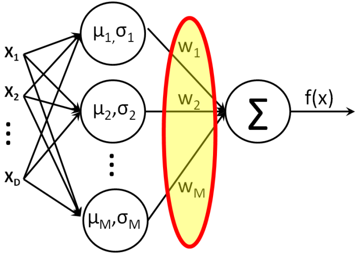

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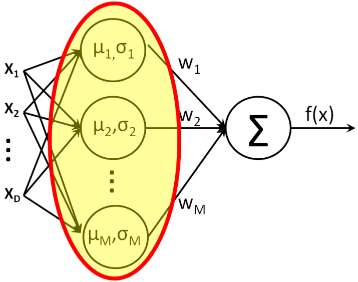

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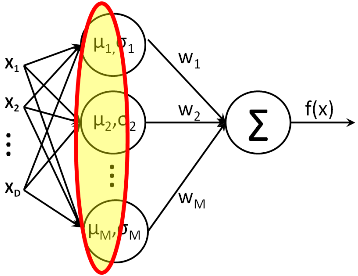

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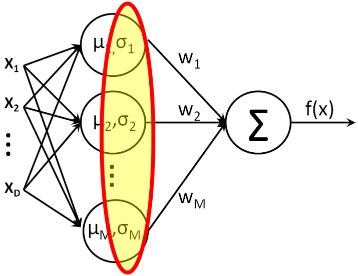

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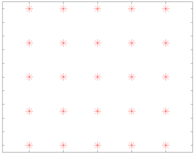
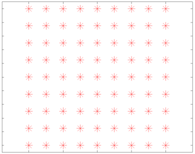
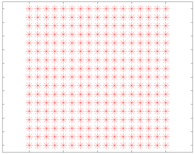
layer l



layer $l+1$



layer $l+2$



Configuration of the parameters

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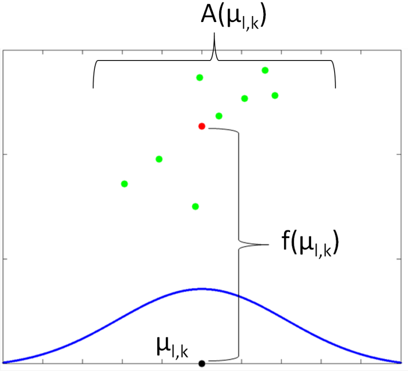
$$f(\mu_{l,k}) = \frac{\sum\limits_{x_m \in A(\mu_{l,k})} z_m \, e^{-\frac{||\mu_{l,k} - x_m||^2}{\sigma_l^2}}}{\sum\limits_{x_m \in A(\mu_{l,k})} e^{-\frac{||\mu_{l,k} - x_m||^2}{\sigma_l^2}}}$$



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Each new layer features half the scale of the previous one:

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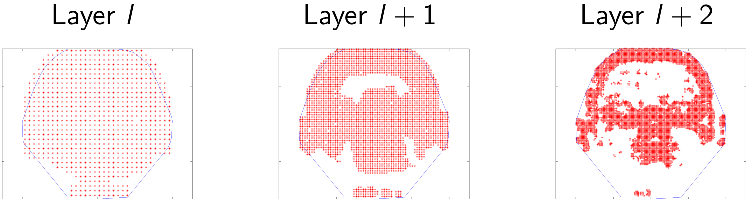
- When $R(\mu_{l,k})$ is over a given threshold, ϵ, the Gaussian is inserted.



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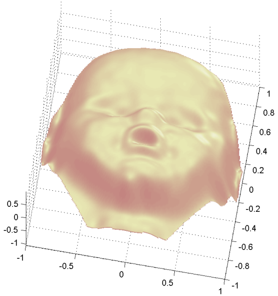
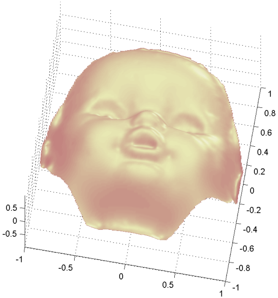
- New layers will be inserted until the training error is under threshold on the entire domain

HRBF vs. RBF



RBF (*newrb* matlab function)
conf time: 776.37 s
units: 341
err mean: 0.0079
err std: 0.0093
rmse: 0.0122



HRBF
conf. time: 16.46 s
units: 6695
err mean: 0.0077
err std: 0.0133
rmse: 0.0154

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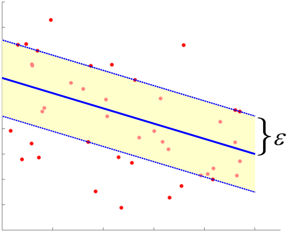

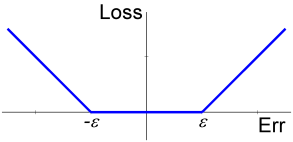

SVR configuration parameters

$$\begin{matrix}\epsilon, C, \sigma \\ X, Z\end{matrix} \longrightarrow \boxed{\text{SVM regression}} \longrightarrow \alpha_i$$

- convex optimization problem
  - unique solution
  - standard numerical sw

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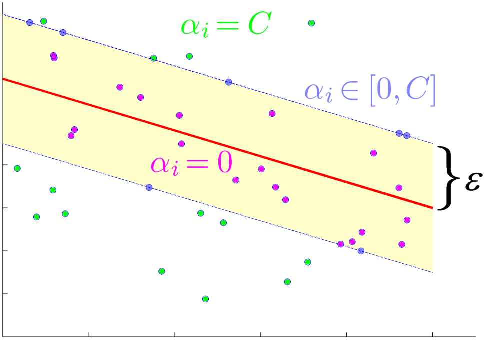

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  - inside: $|\alpha_i| = 0$
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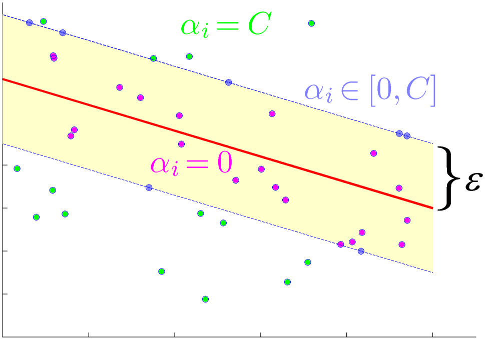

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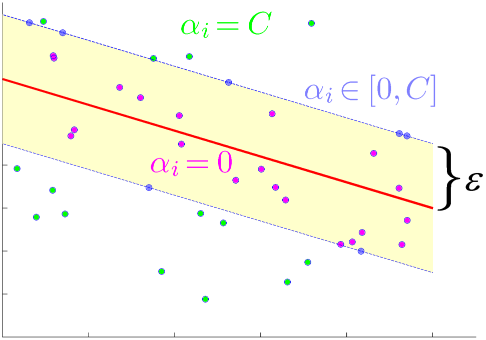

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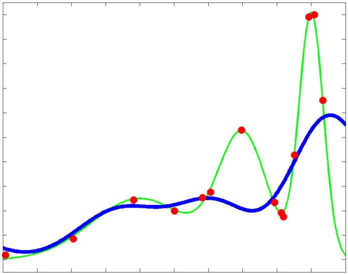
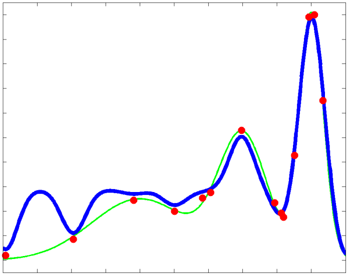


when σ is large



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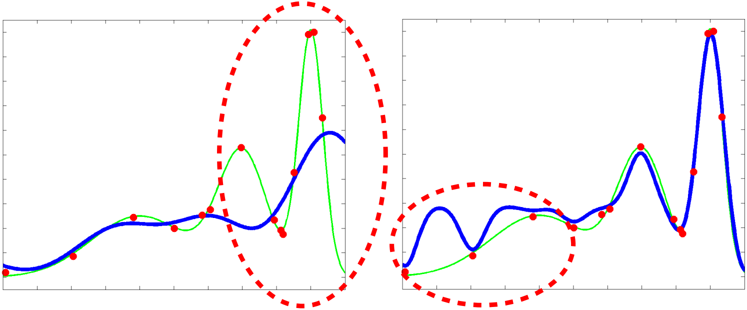

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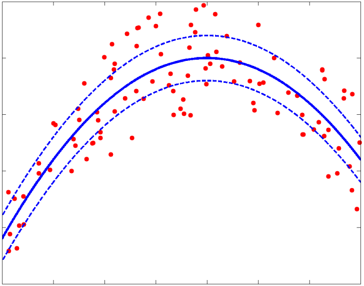
- The only parameter that cannot be estimated from the data set is the parameter ϵ
  - this should be proportional to the accuracy required for the regression

SVs reduction

- The drawback of this scheme is the total number of SVs used, that is significantly higher than in standard SVR
  - large-σ layers have a number of SVs similar to small-σ layers



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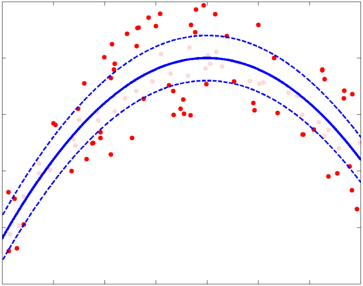

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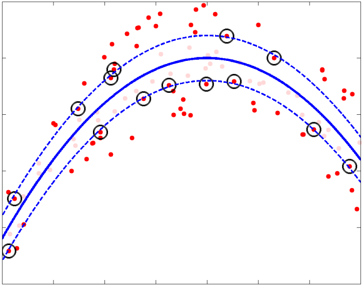

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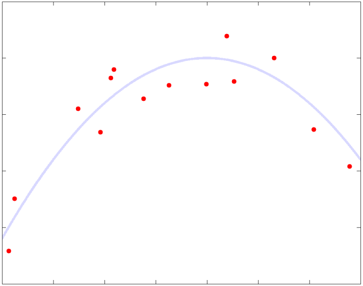

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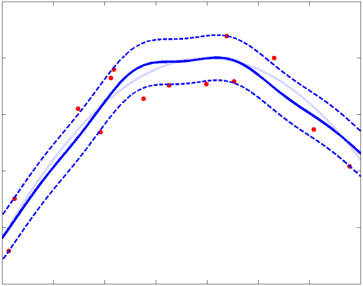

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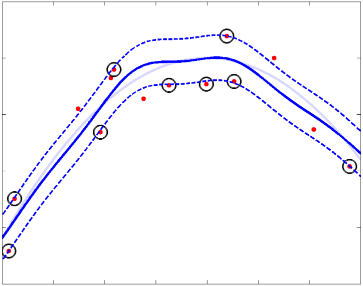

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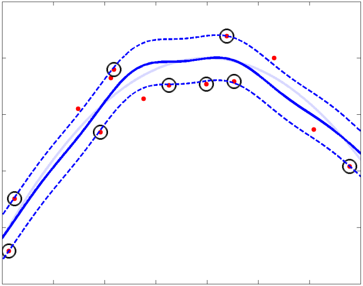

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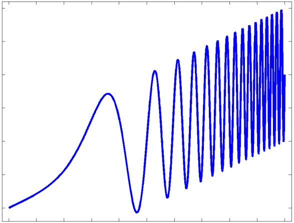
- and used as training set for a second step of configuration
- The so obtained SVs are those of the considered layer

- Due to the hierarchical scheme, the error introduced by the approximation is recovered by the next layers

Results (1)

The training set is a sampling of the function: $\sin(2\pi x^4) + x + u_{[-0.1,0.1]}$

Target function



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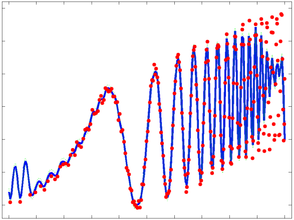
SVR (ε = 0.05, σ = 0.0313, C = 20)



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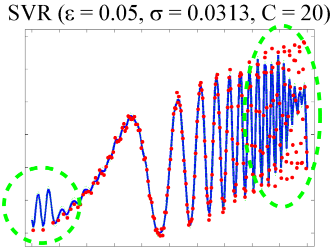

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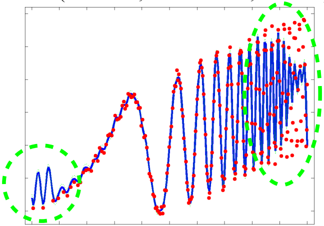
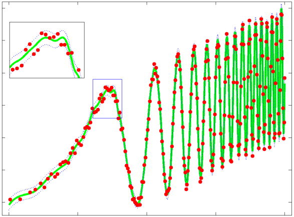
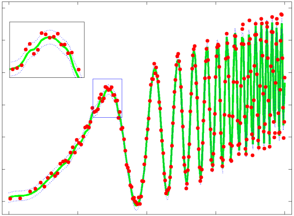


HSVR ($\varepsilon = 0.075$)

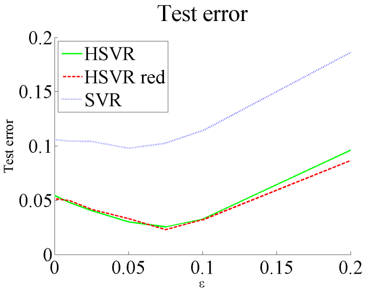


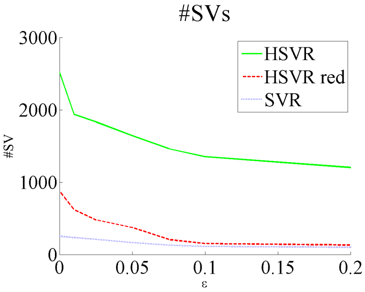
HSVR with SV reduction ($\varepsilon = 0.075$)



	Err_{mean}	#SVs
HSVR	**0.0254**	**1462**
HSVR (red.)	**0.0228**	**206**
SVR	**0.0979**	**163**

Results (2)


Test error
Test error
HSVR
HSVR red
SVR
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
ε


#SVs
#SV
HSVR
HSVR red
SVR
3000
2000
1000
0
0
0.05
0.1
0.15
0.2
ε

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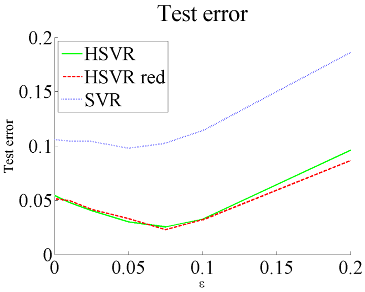
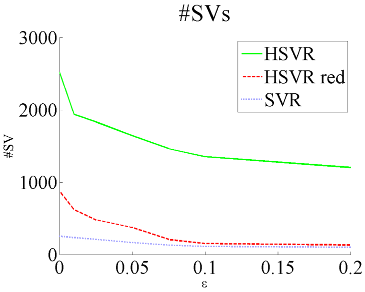
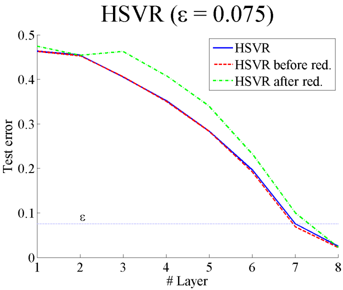
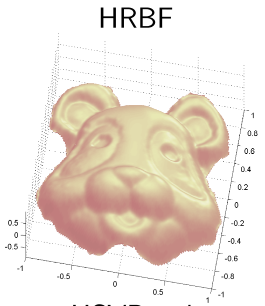


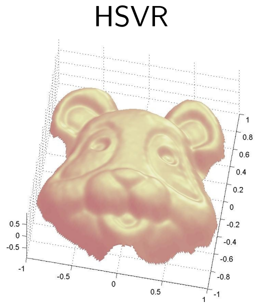


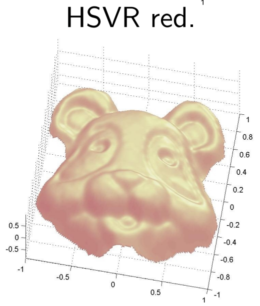


- The error introduced by the reduction step:
  - is recovered in the next layer
  - decrease with adding of new layers

HRBF vs. HSVR



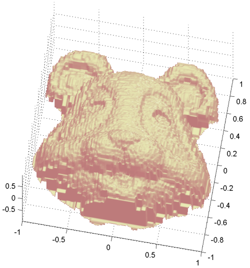




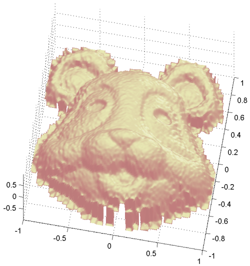
	Err_{mean}	#SVs (tot.)	Time
Online HRBF	0.0112	14,784	44 s
HSVR	0.0110	100,448	682 s
HSVR (Red)	0.0112	11,351	1,104 s

Comparison: other methods

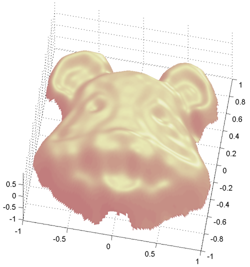
Decision tree

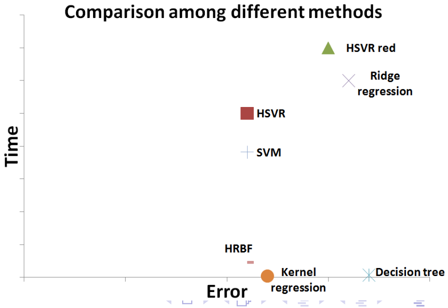


Kernel Regression



Ridge Regression





Outline

The surface reconstruction problem

Computational Intelligence approaches

The regression problem

Neural-based techniques for regression

Why a hierarchical approach?

Radial Basis Function Neural network

Hierarchical RBF model

Support Vector Machine for Regression (SVR)

The Hierarchical SVR model

Conclusions

Conclusions

- Neural-based methods can be effectively applied to regression problems
- Hierarchical structures can improve the accuracy of the standard methods providing:
  - multi-scale solutions
  - more robustness
  - simplification in hyperparameters selection

Conclusions (2)

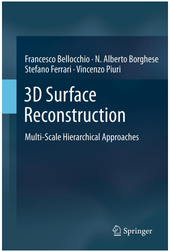
- HRBF
  - Fast configuration based on local operations
  - Few hyperparameters to select
  - Efficient for problems with few input variables
- HSVR
  - More accurated thanks the use of different kernels
  - Hyperparameters space reduced wrt the standard SVM
  - Compact solutions using reduction step

Future directions

- HRBF
  - Real-time implementation
    - technology choice (e.g., CUDA GPU)
    - new algorithmic contraints
  - Extension to non-regular coverage
    - incremental learning to recover the residuals
    - fine-to-coarse iteration
  - Classification
    - smoothness hypothesis
    - non-homogeneous input variables
- HSVR
  - On-line reformulation
    - global optimization
    - memory to store intermediate data

References

Surface scanning and modeling



- F. Bellocchio, N. A. Borghese, S. Ferrari, and V. Piuri, *3D Surface Reconstruction: Multi-Scale Hierarchical Approaches*. Springer-Verlag New York, LLC, 2013.

- F. Bellocchio and S. Ferrari, *Depth Map and 3D Imaging Applications: Algorithms and Technologies*. IGI Global, 2011, ch. 3D Scanner, State of the Art, pp. 451–470.

- N. A. Borghese and S. Ferrari, "A portable modular system for automatic acquisition of 3D objects," *IEEE Trans. on Instrumentation and Measurement*, vol. 49, no. 5, pp. 1128–1136, Oct. 2000.

References (2)

HRBF

- N. A. Borghese and S. Ferrari, “Hierarchical RBF networks and local parameter estimate,” *Neurocomputing*, vol. 19, no. 1–3, pp. 259–283, 1998.
- S. Ferrari, M. Maggioni, and N. A. Borghese, “Multiscale approximation with hierarchical radial basis functions networks,” *IEEE Trans. on Neural Networks*, vol. 15, no. 1, pp. 178–188, Jan. 2004.
- S. Ferrari, I. Frosio, V. Piuri, and N. A. Borghese, “Automatic multiscale meshing through HRBF networks,” *IEEE Trans. on Instrumentation and Measurement*, vol. 54, no. 4, pp. 1463–1470, Aug. 2005.
- S. Ferrari, F. Bellocchio, V. Piuri, and N. A. Borghese, “A hierarchical RBF online learning algorithm for real-time 3-D scanner,” *IEEE Trans. on Neural Networks*, vol. 21, no. 2, pp. 275–285, Feb. 2010.

References (3)

HSVR

- F. Bellocchio, S. Ferrari, V. Piuri, and N. A. Borghese, "Hierarchical approach for multiscale Support Vector Regression," *IEEE Trans. on Neural Networks and Learning Systems*, vol. 23, no. 9, pp. 1448–1460, Sep. 2012.